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ABSTRACT

This case study presents a descriptive portrait of how one general mathematics teacher transformed her classes from a computational to a conceptual orientation. The outcomes of the transformation for student learning and instruction are discussed. The findings suggest student gains in computational competence, mathematical effort, and attitude via concept-oriented instruction exceeded notably the gains of students in other classes where instruction was computationally oriented. The study employed field-research methods including observations, interviews, and the collection of student data across the project's three years to capture and describe the nature and effects of the instructional interventions. At the end of the intervention period, students and instruction were organized in ways that focused student interest, attention, and involvement on the learning of mathematics. Many illustrative segments of classroom interactions are included. Appendices present references on improving communication about mathematics content, social organization of the classrooms to facilitate the learning of mathematics, and modifying the course content of general mathematics to improve learning and teaching. (MNS)

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Research Series No. 172

PAMELA KAYE'S GENERAL MATH CLASS: FROM A COMPUTATIONAL TO A CONCEPTUAL ORIENTATION

Anne Madsen-Nason and Perry E. Lanier

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Anne Madsen-Nason and Perry E. Lanier

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Abstract

In this case study, the authors present a descriptive portrait of how one general mathematics teacher transformed her classes from a computational to a conceptual orientation and discuss the outcomes of the transformation for student learning and instruction. The findings suggest student gains in computational competence, mathematical effort, and attitude via concept-oriented instruction exceeded notably the gains of students in other classes where instruction was computationally oriented.

The teacher was one of three with whom researchers from the IRT's General Mathematics Project collaborated to implement instructional interventions intended to improve learning and instruction in general mathematics classes. The study employed field-research methods including observations, interviews, and the collection of student data across the project's three years to capture and describe the nature and effects of the instructional interventions on her general mathematics classes. At the end of the intervention period, students and instruction were organized in ways that focused student interest, attention, and involvement on the learning of mathematics.

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PAMELA KAYE'S GENERAL MATH CLASS: FROM A COMPUTATIONAL TO A CONCEPTUAL ORIENTATION

Anne Madsen-Nason and Perry Lanier[1]

THE GENERAL MATHEMATICS PROJECT

At a time when our technologically advanced society needs more mathematically competent citizens, our nation's high schools are failing to provide many students with essential mathematics skills. The students who find the study of mathematics unrewarding tend to avoid serious pursuit of the subject. Ninth-grade general math classes, in particular, tend to be unchallenging and disliked by teachers and students alike. Consequently, general math is often both the first and the last high school math course many students take. This study sought to understand why this condition exists and how to find ways to make general math a more successful and rewarding experience.

In the General Mathematics Project "teacher thought and action" and the Carroll factor, "quality of instruction," were considered the context of teacher/researcher collaboration. Teachers and researchers together synthesized literature and designed interventions in mathematics instruction. The question that guided the project was "Can interventions be designed for ninth-grade general mathematics students that concomitantly alleviate constraints and ameliorate learning opportunity and teaching conditions?"

Ninth-grade general mathematics teachers, with notable teaching experience, joined researchers--including mathematics educators, educational anthropologists, and special educators--in addressing the question of "What can be?" (as opposed to "What is?"). Researchers used field-work methods

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[1]Anne Madsen-Nason was a research assistant and Perry E. Lanier a coordinator of the General Mathematics Project. Madsen-Nason currently works with the Middle-Grades Mathematics Project at Michigan State University. Lanier is a professor of teacher education at MSU. Other project coordinators were James Buschman and Linda Patriarca.

(including interviews, classroom observations, and artifact collection) to capture teachers' thoughts about what and why certain ideas, principles, materials, or content were more worthy than others in the context of developing interventions designed to improve mathematics instruction. Similarly teachers' actions in carrying out the interventions with students in general mathematics were captured via classroom observation. Thus, in addition to finding out whether the planned interventions improved the quality of instruction, researchers also became smarter about how teachers adopted and adapted research for practice.

In particular, researchers selected literature to peruse, recommended implementing modified instructional approaches, and provided classroom consultation to alter teachers' thoughts and actions about classroom practice.

Researcher and teacher collaboration led to implementation of three strategic instructional tasks:

1. Modifying the *mathematical content/tasks* that had been typically taught;
2. Increasing the quality and quantity of *communication* about the mathematical content; and,
3. Using the *social organization* to facilitate instruction.

Knowledge concerning the changed thoughts and actions of teachers and the improvement of learning and instruction emerged from the analysis of the data. The results showed that teachers' substantial modifications of these tasks favorably affected students' learning of and attitude toward mathematics.

This case study of Pamela Kaye[2] portrays how modifications of the three strategic instructional tasks transformed her general mathematics class from

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[2]Pamela Kaye is a pseudonym for the general mathematics teacher referred to in this case study.

computationally oriented to conceptually oriented. In highlighting the strategic tasks, we use only a portion of the rich field data we collected. Data for the study were gathered over three school years and therefore involved three different target classes. At the end of each year of the study the first author completed interim analyses of the data that resulted in three case records (Stenhouse, 1978). By design the case records contain only information relative to the project's research questions (i.e., considerable data reduction occurred during preparation of the case records).

The study is presented in three parts. The first part depicts a baseline picture of Kaye's general math instruction that we identify as *computation-oriented* and characterized by fostering students' acceptance of only knowing "how to do a given task" and thinking something significant has been accomplished when the task is completed. Further, we discuss the consequences of a computation orientation including a consideration of the question, "Why would a teacher who enjoys a very good school/district reputation teach general mathematics in this manner?"

The second part conveys the process Pamela Kaye went through in transforming her general math class from one curricular orientation to another. At baseline, the implementation of three instructional tasks were noted: (a) mathematical tasks were selected; (b) these were communicated in some form to students (c) who were organized--intentionally or by default--in some way to complete the tasks. At the end of the project's intervention period the three tasks were still being implemented but differed significantly in their nature and form. This second part of the study traces *the evolution* of the three instructional tasks.

The third part is a portrait of a *conceptually oriented* general math class, one in which students are visibly concerned about understanding the mathematical tasks they are engaged in and feel a sense of confidence and academic accomplishment.

Methodology

In the fall of 1981 four mathematics teachers began collaborating with IRT researchers in an effort to see if ninth-grade general mathematics could be improved and to document carefully the process of transforming "practice then" to the goal of "more successful practice". Three of the teachers--one teacher's school changed to a middle school at the end of year two--participated in this effort through the end of the 1983-84 school year when data collection activities for the project ended.

After these teachers were identified in 1981, their general math instruction was observed, their students' records were examined--as was their respective curricula--and each was interviewed in order to establish a baseline perspective. With these perspectives and the findings and conclusions of earlier inquiries into the nature of the problem in general mathematics, researchers set about ascertaining "What teaching changes would likely address the problem" and "What support measures teachers would likely need to initiate and sustain these changes."

In looking at potential changes, it was first determined that the general mathematics practices of these teachers and conditions in their schools were comparable to findings of earlier work[3] where (a) the curriculum of general

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[3]National Science Foundation Project (SED 79-20593) from 1979-1981 entitled "The Ecology of Failure in Ninth-Grade General Mathematics: An Ethnographic, Experimental, and Psychometric Inquiry" (Perry Lanier, Institute for Research on Teaching, Michigan State University, East Lansing).

mathematics included little or nothing beyond what was studied in grades 6, 7 or 8 and furthermore it was predominantly of a "drill and practice" variety; (b) the setting reduced teachers' concern about general math students' achievement, that is, the teachers' reputation was not jeopardized by student failure/knowledge since this was likely the last math class to be taken by most general math students; (c) the teachers' expectations for assignment completion were observedly low and their time spent on task selection and task explanation was minimal; and (d) the students were less cooperative and had less of a sense of identity with the class than algebra students. Thus the conclusions of the earlier work were judged to be valid for students in the general mathematics classes of these four teachers. The deterrents to students' success in general mathematics were the achievement/attitude history brought to the class; the sense of mathematics' nonrelevance; the resistance to teacher exploration/discussion during lessons; the clamour for mundane but doable drill and practice assignments; the lack of productive school habits (attendance, study, etc.); the patterns of interaction with other students; and the possession of fragmented concepts, alogrithms and problem solving skills.

Based on the above, researchers conjectured that teachers' careful attention to (a) content/task selection (in terms of learning goals), (b) communication of that content to students, and (c) the organization of students when engaging in the tasks would, if unified and orchestrated, provide an intervention to present practice powerful enough to "concomitantly alleviate constraints and ameliorate learning opportunity and teaching conditions in ninth-grade general mathematics classes." These three instructional tasks--content/tasks selection, task communication with and social organization of students--which teachers explicitly/implicitly do with every lesson became

the foci of the intervention strategy. As teachers embarked upon intervention activities they were asked/guided in thinking about changing in terms of these three strategic tasks.

To support teachers as they planned and implemented instruction, that represented a change from usual practice, researchers were theoretically guided by Lewin's change model (Blanchard & Zigarmi, 1981)--unfreezing, changing, refreezing--and Roger and Shoemaker's (1971) deduction that adopted innovations are uniquely adapted by the user. Neither of these theories would be appropriate for a teaching experiment, but in a naturalistic study where teachers/researchers collaborate to bring about and sustain modifications in practice they were particularly useful.

Actual means of support from researchers to teachers, as they changed their practice, took one of two forms--collaborative deliberation on campus and classroom consultation in their respective schools. Both forms were bounded by a consideration of content/task, communication, or social organization.

The deliberation sessions, which began immediately after data collection for the baseline perspectives was completed and continued through May 1984, were attended by all teachers and researchers, including project coordinators and documenters. A typical session consisted of a discussion of one or more readings selected by researchers related to content/communication/organization, individual reports of teachers' efforts to change and their consequences and implications, and group planning relative to the continuing and systematic changes deemed necessary for improvement.

One important purpose of these sessions was to extend the teachers' knowledge. Via discussion of the readings and classroom events, teachers' thoughts regarding tasks/communication/organization were identifiable and

thereby amenable to modification if reflection suggested such a need. For example, deliberations on the selection of content showed that review of whole number operations could occur in the context of work with rational numbers or applications and could therefore be eliminated from the general math curriculum. Similarly, deliberations revealed the necessity of selecting tasks that helped students link mathematical symbols with models--pictured or real. This revelation subsequently led to teachers' learning the significance of discussion in mathematics instruction especially where conceptual understanding is emphasized rather than rules for computation. Eventually it was seen that the organization of students facilitated understanding if the opportunity to discuss tasks was an option. Thus the deliberations based on the readings and classroom trials extended the teachers' understandings of and ways of improving instructional practice in general math classes.

The classroom consultation form of support occurred once a semester for each teacher during the 1982-83 and 1983-84 school years. The pattern that evolved by 1983-84 was consultation on a project-designed fraction unit during the fall semester and an exemplary similarity unit designed by an NSF project[4]--during the spring semester. The consultant met with the individual teacher in preparation for instruction; was present for each day of instruction; provided written feedback on each day's lesson in terms of that day's tasks (as well as suggestions for the next day), communication, and organization; and conferred with the teacher by telephone on the lesson and feedback at the end of the school day. These two forms of support, collaborative deliberation and classroom consultation with their respective component parts--

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[4]National Science Foundation Project (SED 80-18025) from 1980-1982 entitled "Middle Grades Mathematics Project" (Glenda Lappan, Department of Mathematics, Michigan State University, East Lansing).

reading, discussion, cooperative systematic planning and instruction, and feedback--provided teachers with the means of modifying their thoughts and actions and simultaneously provided researchers the opportunity to gather the data to answer their research questions subsumed by the General Math Project's driving question, "Can interventions be designed that will improve learning and teaching in general mathematics?"

Those research questions were as follows:

1. What do teachers see as the central problems in teaching general mathematics? What approaches have they used in dealing with the problems, and what effect do they perceive they have had?

2. How do teachers alter their views about general mathematics as a result of (a) exposure to literature and (b) systematic trial of new approaches to teaching based on that exposure?

3. What concepts, strategies, and research results from the literature are seen by teachers as applicable to the task of improving their general math classes? Through what processes do teachers make sure of new insights and skills?

4. What happens in classrooms when teachers systematically alter their approach to general math? What evidence of student improvement can be found?

To answer these questions, researchers: used the data acquired to determine the baseline perspectives; interviewed teachers periodically across the three school years 1981-82 to 1983-84; observed their instruction regularly--both periodically and intensively; obtained classroom artifacts such as assignments, students' work, test results, grades, and attendance, and interacted with teachers during deliberation sessions and at some social occasions.

Teachers were interviewed individually following the perusal of, but prior to, the discussion of the several readings; at the end of each school year relative to their respective views of general mathematics students,

curriculum, and ways to improve the class; and each semester after instruction on the fraction and similarity units. Interviews were taped, transcribed and stored in the computer for subsequent analysis.

Instruction was observed on a weekly to monthly basis across the three years and on a daily basis during the fraction and similarity units. Researchers took field notes while observing, wrote them up, coded them and stored them in the computer. Field notes were also taken while the consultant and teacher were planning and frequently during the deliberation sessions.

These two major sources of data were augmented with classroom artifacts and the consultant's written feedback. On an annual basis the primary observer for a given teacher would analyze this corpus of data in terms of the research questions. The analysis was preliminary and was used as a means of data reduction. The products of these analyses were case records. Thus by December 1984, a set of three case records had been generated for each of the three teachers participating from Fall 1981 to June 1984.

Case Study Background

After 12 years of classroom experience, Pamela Kaye joined the General Mathematics Project because she wanted to improve general mathematics for herself and her students. Her educational experiences, professional preparation, and interests are in two fields: guidance and counseling and mathematics education. These are reflected in her concerns for the personal and mathematical problems of her students. Kaye works with other elementary, middle school, and high school teachers in her district on professional development matters while maintaining a good rapport with administrators. A Board of Education member said of Kaye, "If there was one teacher in this district I would give merit pay to it would be she."

The students in the high school are mostly from lower middle class (with farming, semiskilled, and unskilled parental occupations making up 75% of the school community). The parents of some students work at a nearby religious-affiliated college and make up the remaining population (with 25% professional, clerical, and skilled occupations). The students are generally compliant and cooperative with school and class rules. Mostly freshmen comprise general math classes although a few sophomores, juniors, and seniors also enroll. Usually these older students are repeating the class to get one of the two credits of mathematics they need for graduation. Most general math students exhibit average ability, and although they do not hate math, they are not particularly interested in studying it.

Arborville High School[5] is the only high school in the district. It has enrollment of approximately 750 students. The school population is classified as 10% rural, 50% small town, and 40% suburban.[6] Essentially all of the students are White (the high school has had a Black family and an Indian family in the last 5 years). Kaye typically teaches two of the four general mathematics classes at the school. Other freshman mathematics classes include Algebra (for more advanced ninth-grade students) and Fundamental Math (for students with learning difficulties).

Kaye's rectangular shaped classroom is typical of most high school math classrooms. Rows of chairs all face the front chalkboard. Her desk is located at the side of the room near the hallway door. Two file cabinets contain ditto masters used in previous lessons and tests. Two metal book shelves

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[5]Pseudonym.

[6]North Central Accreditation Study, 1980-81.

hold student assignments and supplies. Plants hang in two corners of the room. Two bulletin boards contain school announcements, newspaper clippings, and humorous posters.

A Computationally Oriented Class

If they had their druthers, they wouldn't be there.
(Pamela Kaye)

A Typical Day*

The bell sounds at 8:00 and students enter Pamela Kaye's first-period general mathematics classroom. Ms. Kaye is standing near her desk sorting through corrected assignments that are to be returned to the students.

Several students gather around the desk asking to buy or borrow a pencil, use a loaner textbook, or get a piece of paper for the daily review and the assignment.

The students take their seats and chat with one another. Kaye announces from her desk, "Ladies and gentlemen, you have a 100% Quiz on the board. It will be collected as soon as I get the attendance finished."

Kaye frequently gave her students several review problems at the start of each class period. She felt they needed the computational drill and practice and it allowed her the time she needed for record keeping. The Quiz included a review of 10 fraction problems.

The students copy the problems from the chalkboard and begin to compute the answers.

Some students complain to Kaye that they have forgotten how to get the answers, other students look on their neighbor's paper to copy the answers, the remaining students work on their Quiz alone.

Kaye walks to the front of the room and asks the students to hand in their papers.

At 8:10 Kaye begins the daily instruction by telling the students they are to continue working on the assignment given on the previous

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*These are excerpts from project fieldnotes made in Pamela Kaye's general mathematics class.

day which they were unable to finish. She reminds them the assignment includes 124 ratio and proportion problems in their general math book.

Kaye reviews the step-by-step procedure for calculating the answers as she works one sample problem for the students. She then directs the students to the pages with the problems and tells them the assignment will be corrected at the end of the period.

Most of the students start working on the assignment and others have their hands raised for Kaye's help.

During this seatwork period Kaye circulates around the room, systematically walking up and down each of the four aisles. She answers students' questions, checks to see if they are working the problems correctly, keeps them on task, and attempts to make at least one verbal contact with each student.

Some students work together, some students work alone, most students alternate between working alone and working together depending on how difficult they find the problem to work on.

The students who work together frequently socialize about nonmath topics as they work on their assignment.

At 8:45 Kaye announces to the class, "Since most of you are still not finished with this assignment, we will only correct the first half of the problems; I will check the rest myself." She takes her textbook and quickly reads the answers to the problems. She then tells the students to write on their papers their names and the number of problems they missed and hand them forward.

Kaye tells the students they may have the last 10 minutes of the class period to play cards, work on other school work, or to chat quietly. Most students spend this time socializing with each other.

At 9:00 Kaye dismisses the students as the bell rings.

The initial observations of Kaye's general mathematics class indicated that an average period consisted of 5 minutes of nonmath-related management and socializing activities at the start of the period, 9 minutes of whole-class instruction on the content of the daily lesson, 30 minutes of seatwork (from the textbook or a worksheet), 1 minute of checking the answers to the daily assignment, and 15 minutes of nonmathematical activities (5 minutes at the start of the class and 10 at the end which included managing papers,

keeping records, and socializing). The students expected a 5-minute review quiz at the start of the period followed by a "here's how to do these" presentation of the daily content, a drill and practice seatwork assignment, and some free time at the end of the period.

Communication During Instruction

> They weren't that interested in understanding the math; they wanted to know what to do so they could get on with it.
> (Pamela Kaye)

The observations of Kaye's general mathematics class indicated that communication about the math content was sparse and minimal. Kaye did not use explicit mathematical language during daily whole-class instruction. For example, numerators and denominators were referred to as "top and bottom numbers," cubes were called "boxes," and decimal fractions such as three-hundredths became "point zero three." Combined with this use of nonmathematical language were other communication patterns that characterized computationally oriented instruction such as the teacher's explanation of how to work the problems on the daily assignment (as opposed to providing an explanation of why the mathematical steps work) and emphasis on memorizing computational rules.

Kaye believed the students were more interested in knowing how to work the problems on the assignment so they could finish than they were in understanding the mathematical concepts. To let the students get on with their task, Kaye reduced instructional communication to a set of directions or instructions.

An avoidance of mathematical terms typified the communication between Kaye and her general math students. The following selection from the field notes describes the typical communication pattern between Kaye and her students.

Kaye tells the class that the easiest thing to do is to find out how many eighths there are, then to add the top numbers together. She continues:

Kaye: What do you have?

John: One and eleven-eighths.

Kaye: What is wrong with the problem?

Class: It's top heavy!

Kaye changes the answer to two and three-eighths.

Direct instruction included Kaye's use of nonmath language as she demonstrated the procedures students should follow for working the problems. The following observation illustrates her "how to do this" instruction for adding unlike fractions.

Kaye asks: What about this one?

$$\begin{array}{r} +\ \frac{5}{8} \\ + \\ -\ \frac{9}{16} \\ \hline \end{array}$$

Charles: You have got to get a common denominator first.

Kaye: That's the easiest way to solve the problem. Then you have to decide on the sign and subtract.

Robert: How did you get that answer?

Kaye writes the following problem on the board:

$$\begin{array}{cc} +\frac{1}{6} & +\frac{1}{6} \\ + & \\ -\frac{2}{3} & -\frac{4}{6} \\ \hline \end{array}$$

Kaye: You have one-sixth and negative two-thirds so you get a common denominator which is six and the two-thirds changes to negative four-sixths. Since the signs are not the same you subtract and take the sign of the larger number.

Kaye thought the students would be less confused and ask fewer questions when working on their assignments if she simply told them the procedures for calculating the answers in nonmathematical language. During a review of addition of decimal integers, Kaye was observed telling the students how to determine which of two decimal values was the larger.

Kaye: The problems you have been having are mistakes that you are making because you are not able to tell which number is bigger. I want you to put a check mark on the number in each pair of numbers you think is the biggest.

Kaye has written the following pairs of numbers on the chalkboard:

(1)	(2)	(3)	(4)	(5)
+.6	+.7	+.05	+.05	+.45
-.09	-.04	-.2	-.6	-.2

Kaye walks around the room checking the students as they are working.

Kaye: Are most of you done? Now, what I want you to do is to fill in all the empty parts with *0s*. The easiest way to tell which number is bigger is to fill in the same number of *0s* and take the decimal out and then just compare the numbers.

In addition to an emphasis on nonmathematical language and the demonstration of computational procedures, another frequently observed characteristic of the pattern of classroom communication was an emphasis on student memorization of algorithmic rules. In the following observation, Kaye illustrated the rules for adding integers using a numberline and then told the students to copy the rules for adding integers because she believed that students would remember the rules longer if they wrote them.

Kaye: I want you to write this down on your papers (writes on the board).

<u>Adding Signed Numbers</u>

1. If the signs are the same, add and put on the sign.
2. If the signs are *different*, find their *difference* [*subtract*] and put in the sign of the "largest" [*the most*].

Kaye: I am going to write down "different" and "difference" which means to subtract. If the signs are different you

have to go so many to the left or right on the numberline and then you have to go back. If you have a positive four and a negative one what would you do?

Stanley: Five, no positive three. Cause you go four to the positive way and then you would come back one.

Kaye: Yes, but I want it stated in the way of the rule.

In another observation Kaye was asked by a student to re-explain how she had obtained an answer. Kaye responded to his request by restating the rule for adding integers.

Kaye demonstrates how to solve the following problem:

$$+\frac{2}{3} = +\frac{8}{12}$$

$$+$$

$$-\frac{3}{4} = -\frac{9}{12}$$

Kaye: Your answer is negative one-twelfth.

Stanley frowns at her answer.

Kaye: You are frowning, Stanley. What is the trouble?

Stanley: I don't get it, I added.

Kaye: When you have unlike signs you have to subtract. When you have like signs, then you add.

The nonmathematical language, the "here's how to do these" explanations, and the focus on algorithmic rules were three communication strategies Kaye used to reduce the amount of time the students spent in whole-class instruction. Reducing the time spent in direct instruction left more time for the seatwork drill and practice assignments.

Mathematical Tasks During Seatwork

> Drill and practice exercises are needed to cement it into their little minds.
>
> (Pamela Kaye)

Seatwork consumed the major portion of time in Pamela Kaye's general mathematics classes. The 30-minute seatwork assignments were characterized by drill-and-practice exercises, student on- and off-task behavior, and Kaye's working individually with students. Kaye thought drill and practice exercises served two purposes: first, to let her know if the students understood what was going on and second, "to cement it into their little minds." She believed that without sufficient drill and practice the students would not remember, "no matter how much they understood it the first time." Kaye's emphasis on drill and practice was evidenced throughout the observations. The students were frequently given textbook or worksheet assignments containing numerous and repetitive computational problems. At the beginning of one class period Kaye made the following announcement to the students regarding the length of the assignment they would be given:

> I have a lengthy assignment for you and it will probably take you two days. We are moving into something that will be kind of hard and that's why I want you to do all of these problems.
>
> Please copy the problems down. This is a long assignment and I debated whether or not to give it to you in its entirety because I thought it might blow you away. I do want you to do all of these problems on page 189.
>
> There are some decimals in these problems. I want you to try them. I am not going to give you anymore help on these now. Just try them. I don't want to do anymore with you here. I will be around to work with you.

The lengthy drill and practice seatwork assignments allowed Kaye to work with students individually while the rest of the class worked on the task. During an interview Kaye said,

> If I could be totally idealistic, I would teach every kid one-to-one. I really think I do my most effective teaching when I can sit down with the kid and get immediate feedback from him or her, and find out where the gaps are and go from there.

The students who had difficulty understanding the assignment received Kaye's help while the rest of the class worked alone or in small groups. The drill-and-practice computational assignments were routine and simple enough to permit the students to chat and socialize with one another as they worked. As the seatwork time progressed and the students completed the assignment, socializing increased in frequency and volume. During the seatwork periods, the observer noted frequent increases in student talking or socializing as they finished the daily assignment. This increased noise level caused Kaye to stop working individually with a student and ask the noisy students to quiet down. As she went back to work with the student other groups of students would begin socializing when they completed their assignment. The following example typifies the interruptions.

> The noise level continues to increase with many groups of students now talking.
>
> Kaye: Quiet down, gentlemen. (Kaye continues working with Roger.)
>
> Pete and Joe are on task and working with each other. Jim is watching them. They start talking about the worksheet puzzle they are working on. It is a map of a trip from Detroit to Boston.
>
> Pete: Well, have you ever been to Boston before?
>
> Joe: No, but I went to Pennsylvania.
>
> Jim: It would be neat to go by motorcycle!
>
> Kaye reminds the second group of students to be quiet as another group is heard talking about trips they have taken.

The routine drill and practice exercises could easily be completed by most of the students while they chatted with each other on other nonmath topics. One student was observed talking to her neighbor during the seatwork time about a movie she had just seen. She turned to a classmate behind her and said, "I'm working on my math, too. It just helps when I talk."

Kaye used routine assignments during seatwork to give the general math students the computational practice she believed they needed. The seatwork assignments were lengthy but easy enough to be successfully completed by most of the students with minimal effort. She wanted the students to be able to do their seatwork assignments correctly and with ease because she believed they would be "fooled" into thinking they were successful in mathematics. Kaye thought this false sense of mathematical success would lead to an improved mathematical attitude.

Interviews and observations indicated evidence that Kaye's class was indeed computationally oriented. She wanted her general mathematics students to attain computational competence. She believed the more time the students spent practicing the algorithms for manipulating whole numbers, fractions, and decimals, the better their skills would be. She limited the whole-class direct instruction to a few minutes of demonstration on how to work the problems on the daily assignment. Though she personally found mathematical ideas and concepts interesting, she did not consider them as vital to the survival of her students as computational skills; therefore, she gave the former little or no attention, letting the latter drive the instruction. The emphasis on computational competence limited the communication between Kaye and the students, established minimal levels of achievement and performance, and fostered an environment that was mathematically unchallenging and intellectually unrewarding for both Kaye and her students.

Thoughts About Improving the General Math Class

> There's not the excitement. I'm not sure they're really getting all that much out of it, because they're not that interested.
>
> (Pamela Kaye)

Kaye said that when she heard the term "general math" she thought about basic computation first. To her, the general math curriculum included addition, subtraction, multiplication, and division of whole numbers, decimals, and fractions and some work with percents. She expressed great frustration with the general mathematics curriculum as it was and believed that changes could occur only with increased knowledge and use of other materials on her part. Kaye was concerned about students' poor attitudes toward mathematics--not only did they not understand it but they found little use for it. She believed if the students could see math as relevant, if they could see the need for it, "then it wouldn't be nearly the struggle to teach them." She said, "I'm afraid that too much of what we do in math doesn't really mean anything to them." Kaye thought there was a motivational factor involved in real-world kinds of experiences for the students, "The more you show them how things have an actual application, the more interested they will be in what they are doing."

When Kaye was asked to think of any changes that might improve general mathematics, she said she thought there should be more use made of manipulable materials and hands-on, real-world types of experiences. She wanted to have less paper-and-pencil work but wasn't certain how this could be accomplished with ninth graders. She wanted to find ways to get the students talking about their concerns with mathematics and to get them to interact with one another about math.

When Kaye was asked to describe the perfect general mathematics class she replied,

> Students who are excited about working, or at least more than just tolerant. That first-hour class tolerates me, they're not that excited, but they're not going to hassle me about it. For them, it's, I'm here. I've got to be here.

There's not the excitement, and I'm not sure that they're really getting all that much out of it, because they're not that interested.

If they had their druthers they wouldn't be there. It has nothing to do with me, it's just that it's general math class.

Discussion

Three questions emerged from the analysis of the baseline observational and interview data. Answers to the questions are necessary to understand Kaye's general mathematics instruction and the consequences thereof for herself and the students. The following is an attempt to answer these questions from our perspectives as Kaye's observer and consultant.

1. What factors led Kaye to create a computationally oriented class and what were the consequences for learning and instruction?

Kaye's decision to emphasize computation emanated from her knowledge of herself as a mathematics teacher, of the students as learners, and of the content of the mathematics curriculum. She acquired this working knowledge from her preparation as a secondary mathematics teacher, her years as a general mathematics teacher, and the concerns she had for students and their problems. These experiences provided a basis for the creation of an educational environment which, for Kaye, optimized learning and instruction in general mathematics. Her preparation as a secondary mathematics teacher emphasized the teaching of geometry, algebra, trigonometry, calculus, and higher mathematics; she never studied the teaching of general mathematics. Her beliefs about general mathematics were similar to those of her mathematics teacher educators and colleagues as well as the textbook writers and publishers: (a) the content of general mathematics was computation (b) instruction was the demonstration of step-by-step procedures, and (c) learning to compute was acquired through drill and practice. This set of beliefs, her apparent lack of knowledge and understanding about how mathematics was learned and how

mathematical misconceptions developed (which fostered children's problems with learning mathematics) led her to rely on the expertise of those who purportedly knew how to develop the most appropriate general mathematics curriculum.

Kaye was confident she could teach general mathematics students how to compute--all she had to do was review the step-by-step procedures contained in the textbook and then let them work on similar problems. She was not confident in her ability to show these students the real-life applications of the assigned problems. She wanted to give her students reasons to justify why they needed to master computational procedures, but her lack of vocational experiences combined with a largely theoretical mathematical background prevented her from being able to do so. Consequently, she was discontented with giving her students countless drill-and-practice exercises and yet was unable to provide them an alternative nor with a reasonable rationale for such assignments.

Further, her admitted failed attempts at teaching mathematical concepts along with the inability to teach mathematical applications led her to focus on the drill and practice of computational skills. She justified this by believing that students' learning of mathematical concepts and applications would follow the mastering of the computations of these problems (e.g., once they could compute percents she could teach them the underlying concepts and related real-life applications). The students, however, never became successful at computing percents (because they did not understand the concepts which provided the foundation for the algorithms) and since they had not, Kaye did not teach them the percent concepts, let alone the applications.

Such a set of beliefs about learning and instruction spawned the creation of a computationally oriented classroom for general mathematics students--students who are in general mathematics classes because they aren't in algebra

classes. They are not in algebra classes because they do not want to be, not necessarily because they lack the mathematical ability. There were students who saw little use for mathematics and showed little interest in learning math. Also, they usually had a history of unsuccessful mathematics experiences. While some mathematics teachers characterize general math students as lazy and slow, Kaye characterized them as lacking sufficient self-confidence to be mathematically successful. Regardless of view, the consequences are lower expectations by teachers for general mathematics students than for other math students. A consequence of Kaye's assessment was her creation of a classroom environment where the students would feel comfortable and not threatened by either her or the mathematics. In creating such an environment she removed any opportunity for students to experience the dissonance between what they knew and what they knew they did not know and needed/wanted to learn. The opportunity for mathematical growth and development had been eliminated in this class where low expectations prevailed.

While certainly other factors influenced Kaye's beliefs about and practice of teaching general mathematics, it was primarily her mathematical and professional studies plus her experience in teaching mathematics which provided the motivation for creating this computationally oriented general math class. This conclusion gave rise to the question, "How did Kaye translate her beliefs and thoughts into instructional strategies enacted in the classroom?"

2. What instructional tasks/strategies did Kaye implement that fostered a computationally oriented class?

Instructional tasks/strategies are planned and purposive; they evolve from the knowledge and beliefs teachers have about themselves, their students, and the mathematical content. Kaye's instructional tasks/strategies were compatible with her beliefs about what content students should learn and how they

could best learn it: They typified the tasks/strategies implemented in computationally oriented classes in which the production of answers to routine problems is emphasized.

Kaye used the instructional task/strategy of demonstrating how to calculate an answer for her students by emphasizing a step-by-step procedure for the calculation followed by an assignment consisting of a set of similar problems. Because she evaluated the seatwork assignments by counting the number of completed problems, the emphasis was neither on the quality of the answer nor the thoughts of the students as they worked on the solution. This demonstration strategy limited the mathematical communication between Kaye and her students. Kaye felt that questions, discussions, and explanations took time away from the seatwork period needed by the students for drill and practice. The emphasis on computational procedures prohibited students from making conceptual linkages across mathematical content areas and engaging in solving challenging or interesting mathematical problems.

Another instructional task/strategy used by Kaye was that of teaching parts of multistep problems as separate lessons because she believed students could handle the one- or two-step calculations with some success. If students completed a series of lessons where parts to problems were computed, she reasoned, they would be able to put the parts together and successfully work the multistep problem. For example, students practiced reducing fractions to lowest terms several days before they were given problems that required answers in lowest terms. This was one of several fraction skills practiced in isolation.

Kaye's instructional tasks/strategies for getting students to work the daily assignment communicated to them that completing the task was far more important than learning the mathematics. First, she rewarded her students

with free time upon completion of the daily assignment. Consequently, many students quickly learned to share answers in order to finish early enough to play a game of checkers or cards. Second, Kaye gave students credit for simply working on the assignment. She told them no one would fail in her class as long as they just tried to do the work. The students soon realized that with only minimal effort they would pass the class.

Kaye's instructional tasks/strategies typified those previously observed in computationally oriented classes. Direct instruction consisted of the demonstration of the solution to a sample problem. Seatwork assignments were easily completed with minimal thought and effort and students who finished early were rewarded with free time. Grades reflected the number of problems completed rather than the students' mathematical learning.

The *learning* of mathematics was not seen as a goal since it was not rewarded--the reward was to finish so one could engage in a game. Second, Kaye gave the students credit for simply working on their daily assignment. She let them know that no one would fail in the class if he/she just tried to do the work. Consequently, the students realized that for minimal effort, to say nothing about the quality of work, they would pass. In fact, by her own admission, she encouraged it--rewarded it, by passing those students. Her expectations for the students' low achievement were evidenced in her grading and evaluation of their work.

Kaye's instructional strategies typified those previously observed in classes with a computational orientation. The amount of time spent in whole-class instruction was limited to a demonstration of how to work a sample problem. This limited the quality and quantity of content communication between her and her students. The seatwork assignments were so routine that they were easily completed within the seatwork period by nearly all the

students with minimal thought and effort. Students were encouraged to stay on task and complete their work. Those students who finished early were rewarded by free time and not having to work more mathematics problems. Grades were used as incentives for task completion rather than as a measure of mathematical learning.

3. What were the outcomes of computationally oriented instruction for Kaye and her students?

Four beliefs about what teachers should do undergirded Kaye's general mathematics instruction: (a) lead students to computational competence, (b) keep students on task and productive, (c) teach students to be cooperative, and (d) make students feel comfortable in the mathematics class. These beliefs and the tasks Kaye implemented to lead students to computational competence resulted in students spending most of the class period practicing computations similar to those practiced in previous mathematics classes, practicing procedures versus learning mathematics, and having no opportunity to experience different mathematical topics (such as probability and statistics) which link arithmetical computations to related mathematical concepts, principles, etc. Rather than think about and create new lessons Kaye relied on textbook reviews and assorted drill- and-practice worksheets to determine the content and instruction.

The instructional tasks/strategies Kaye used to keep students working and productive resulted in the students mechanically laboring through sets of specified procedures to get answers. Both Kaye and the students knew these drill-and-practice assignments were mundane, yet an unstated, common understanding existed: This was the work that general mathematics teachers gave and that general math students did. Filling the allotted drill-and-practice period with work required Kaye to assign a great deal of problems, which

resulted in numerous papers to be corrected and scores to be tabulated. Kaye was overwhelmed with collecting, grading, and returning math papers.

One instructional task Kaye used to promote student cooperation included allowing students to work together. One method she used to promote student cooperation was letting students play games when their daily assignment was finished. Although this method fostered student cooperation it did not promote the students' learning of mathematics. In their drive to complete their work, students shared or copied answers and socialized as they worked. When students reviewed with each other the procedures for calculating the answers, they frequently acquired additional misconceptions. Because students socialized as they work together, Kaye spent a considerable amount of time during the seatwork period quieting students, keeping them on task, monitoring their copying, and re-explaining correct computational procedures to pairs or groups of students who had made errors in their work.

Kaye's selection of instructional tasks/strategies that she felt enabled students to feel comfortable in mathematics class had the greatest negative outcome for the students. Her creation of a classroom atmosphere that was comfortable and nonthreatening communicated, albeit inadvertently, to the students the low expectations she had for their ability to learn and achieve success in mathematics. This environment emphasized nonacademic instruction, encouraged low performance levels, and did not hold the students accountable for learning the mathematical content. The students responded to Kaye's low standards and expectations with minimal performance and interest in mathematics. Kaye's beliefs about what tasks would promote teaching and learning in general mathematics had three outcomes. First, her emphasis on practicing basic computational skills allowed little time in the curriculum for the

development of substantive mathematical concepts, experiences, or understandings. Second, her belief that general mathematics students were limited in their ability to be successful in mathematics, were uninterested in learning anything new in mathematics, and were not excited or challenged by mathematics caused her not to invest the time in planning or developing math units that could have been meaningful and challenging. Third, Kaye's emphasis on establishing a nonthreatening and comfortable environment (which she believed would contribute to increasing student confidence) compromised her role as an effective mathematics teacher and promoted meaningless busywork.

The Evolution From a Computational to a Conceptual Orientation

Pamela Kaye's computationally oriented class is best characterized by what it was not:

1. It was not a class where the learning of mathematical concepts was emphasized.
2. It was not a place where mathematics was talked about.
3. It was not an environment that fostered the desired learning and teaching of mathematics.

Modifying this computationally oriented class required an instructional evolution. The mathematical content would have to evolve from its computational focus to a conceptual orientation. The quality and quantity of classroom communication would have to evolve from minimal giving of directions and procedures to substantive mathematical dialogues enriched by questioning, explaining, and discussing. The social organization (the arrangement of students, routines, and procedures) would have to evolve from a mode that promoted the mass production of answers to one that enhanced and encouraged the

development of mathematical thinking and understanding. Modifying these three strategic instructional tasks is central to improving general mathematics classes.

Throughout the evolution important links had to be established within and across each of three strategic instructional tasks: content, communication, and social organization. Classroom instances and interview segments are used to highlight these linkages and describe the evolution of Kaye's instruction from a computational to a conceptual orientation. Although modifications of the three instructional tasks occurred interactively and somewhat simultaneously, they are portrayed serially.

The Evolution of Mathematical Content/Tasks

> You're still working with add, subtract, multiply, and divide in the whole numbers, the decimals, and the fractions--but it's with a conceptual understanding rather than computational.
>
> Everything we do has led me to believe that the major problem with students is they don't have any understanding of what's going on.
> (Pamela Kaye)

Although computational competence remained a major goal, Kaye transformed the mathematical content and tasks of general mathematics from being computationally to conceptually oriented in order to attain this goal. This transformation is characterized as an evolution of mathematical content. Evolution is defined as a process of continuous change from a lower, simpler, or worse to a higher, more complex, or better state. The description of Kaye's computational class portrays the "lower, simpler, or worse" state of mathematics teaching and learning. The content and tasks of the computationally oriented class provided the primordial substance from which evolved the content and tasks of the conceptually oriented class, "a higher, more complex, or better state."

The process of content evolution was sparked by Kaye's project-related experiences which included reading and reviewing selected literature related to improving learning and instruction in mathematics, collaborating with the project's teachers and researchers, and implementing modifications of strategic instructional tasks intended to improve the general mathematics class. Her selected instructional tasks promoted the establishment of conceptual linkages across different mathematics topics as well as links between new and previously taught mathematical content.

Linkages across mathematics topics. The instructional modifications Kaye implemented to teach concepts about fractions were also used in the decimal and percent units. She continuously sought ways to link the concepts of fraction, decimal, and percent together because she believed this would help the students gain a better understanding of the mathematical relationships that existed between them. The first vignette illustrates instructional modifications Kaye made in a unit on fraction concepts.

On the chalkboard the following is written:

three-fourths

Kaye: Let's find out what you know. Mr. Biggs, what does that say?

Richard: Three-fourths.

Kaye: Have you seen it written any other way?

Richard: You could have four squares and three of them colored in. That would be three-fourths.

Kaye draws on the chalkboard:

Kaye: Does anyone see another way?

Kenneth: Draw three, put a line under it, then put a four under that.

Kaye writes on the chalkboard:

$$\frac{3}{4}$$

Kaye: Is there another way?

Kenneth: Draw a circle with three-fourths of it colored in.

Kaye draws the following:

Kaye: Like this?

Kenneth: No, it had part of it shaded in.

Kaye: Like this?

Kaye draws the following:

Kenneth: No, do a line cutting across it and then draw another line cutting it down.

Kaye draws the following:

Kenneth: No, like this. . . straight! Straight up. Make the line straight up and then make the other line straight across!

Kaye draws the following:

Kaye: All right, is there another way?

Richard: Probably.

Kevin: You could make a square and put a line in vertically and then another one going across.

Kaye draws the following:

Kaye: All right, which one is vertical?

Kenneth: It's an up and down line.

Kaye: Well then, what's horizontal?

A student: It's a line going across.

Kaye writes the words "vertical" and "horizontal" on the drawing of the square:

Kaye: Do all the parts have to be equal?

The students: No.

Kenneth: No, the parts don't have to be equal--it just makes it neater.

Kaye: All right, if I do this...is this three-fourths?

Kaye draws the following on the chalkboard:

Kenneth: No, that's three and a half.

Kaye: What about if I had done this?

Kaye draws four lines of equal length:

Mary: Put a circle around three of them.

Kaye circles three lines:

Kaye: What about if I had done my circle like this?

Kaye draws the following:

Kaye: And what if I had shaded it in like this?

Kaye shades in the circle:

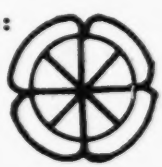
Shara: They're not equal.

Kenneth: Well you could put in extra lines like this. And that would make them equal.

Kaye draws the following:

Kenneth: Now, you could shade in every two of them.

Kaye marks off every two pieces:



Randy: It looks just like a flower.

Kenneth: You could make one of them squares and just shade in one of them and then the blank ones would be three-fourths.

Kaye draws the following:

Kaye: Have you seen these words before?

Kaye writes on the chalkboard:

$\frac{1}{2}$ numerator
denominator

Randy: Yes, last year our teacher told us once.

Kaye: Then I could say there are two pieces in the whole circle and the word *whole* is the one I want to emphasize. If you have a circle that means you have one piece in the whole. What would it look like if you had two over one?

Kenneth: You would need two circles.

Kaye: I would like you to put this on your papers.

Kaye writes the following on the chalkboard:

numerator $\frac{1}{2}$	1 part shaded
denominator	2 pieces in the whole

Kaye: If I had four parts shaded out of four parts what would I have?

Kaye writes on the chalkboard the following: $\frac{4}{4} = 1$

Richard: You would have all of them shaded, that would equal one.

Kaye: How did you know that equalled one?

Randy: You would divide.

Kaye: If you didn't shade any of them what would you have?

A student: Zero fourths.

Kaye writes on the chalkboard: $\frac{0}{4}$

Kaye: All right, we are going to look at part of the line and we are going to call this a numberline with these segments.

Kaye draws a numberline:

Kaye: How are you going to show three-fourths?

Kenneth: Put in four lines between the zero and one.

Kaye draws in the lines on the numberline:

Some students: No! You just need three lines!

Kaye changes the number of lines she has drawn to three:

The students: No, not like that!

Kaye: All right Melanie, what would you do?

Melanie: Make four spaces and then shade in three of them.

Kaye changes her drawing:

Kaye: All right, again someone has shown us where a lot of mistakes occur. What Kenneth did was count all the points on the line like this.

Kaye puts points over the lines on the numberline:

Kaye: And there are four points on that line, but you need to have four segments, four line segments.

Kenneth: Give us an illustration of that, Ms. Kaye.

Kaye: Alright class, what is a fraction?

The students: Parts to a whole.

Kaye: What are the parts?

The students: Numerators and denominators.

The strategic instructional task of changing the mathematical content/tasks had been modified through techniques that encouraged students to develop a conceptual understanding of fractions: (a) pictorial representations, (b) verbal descriptions, (c) counter examples, (d) student-generated examples, (e) brainstorming ("What does a fraction mean?"), and (f) comparing the part to the whole. These techniques were continually used throughout the unit and gave the students a variety of ways to think about and understand fractions.

When Kaye moved from fractional concepts to the computation of fractions these same techniques were used. The following observation describes the beginning of a lesson on multiplication of fractions. Kaye asked the students to work with her on their papers (in controlled practice) as they discussed the meaning of multiplication of fractions. The lesson began by asking the students to describe how they thought about multiplication of fractions. She then moved the students from this computational focus into the conceptual domain where she used pictures to describe multiplication of fractions.

Kaye: I want you to look up here. What could you do for this one?

Kaye writes on the chalkboard: $8 \times \frac{1}{2}$

Dick: Put 8 over 1 and then put times 1 over 2 and now I want to get a common denominator...no...All right, cross out the 2 and make it a 1 and cross out the 8 and make it a 4.

Kaye writes the problem as Dick talks:

$$\frac{4}{1} \times \frac{1}{1}$$

Kaye: And now what?

Dick: So 4 times 1 is 4 and 1 times 1 is 1.

Kaye: So 4 over 1 is what?

Dick: 4 wholes.

Kaye: Mr. Jones, what could you do to draw a picture of 8 times a half?

Larry: Draw 8 circles.

Kaye draws eight circles on the chalkboard:

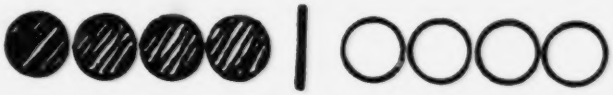
Kaye: Tom, why did Larry draw 8 circles? (Tom doesn't answer.)

Kaye: Can anyone help him out?

Dick: Because 8 means 8 wholes.

Kaye: All right Tom, we have a group of 8 things and you take half of 8 which is four.

Kaye draws on the chalkboard:



Kaye: Is there another way?

Gene: Take 4 circles and cut them in half and you would have 8 halves.

Kaye draws the following:

Sue: You could take 8 circles and cut each one in half and just take half of each circle.

Kaye draws the following on the chalkboard:

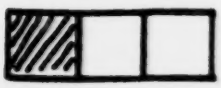
Kaye: That's the way I would like you to see this multiplication. Now, I want you to write one-half times one-third on your papers.

Jim: You have to get a common denominator now.

Sally: No, you don't.

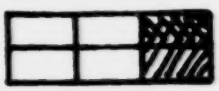
Kaye: Divide a rectangle into thirds like this.

Kaye draws the following:



Kaye: I want you to look at the third--shade it in. Now, cut the third that you have into half and shade it in.

Kaye shades in the third first then the half of a third second:



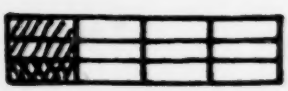
Kaye: Multiplication means you have groups of one-half of a third and now I want you to tell me what you have.

The students: One-sixth.

Kaye: Write this one, please. One-third times one-fourth. I want you to start with one-fourth of a rectangle and shade it in. Then I want you to take one-third of that fourth.

The students start working while Ms. Kaye monitors their answers.

Kaye writes the result on the chalkboard:



Kaye: That's good. One-fourth cut into thirds tells you to take one-third of a fourth and you need to find out how many equal pieces you have.

Sally: So, you have 12 equal pieces and one-twelfth of that is shaded.

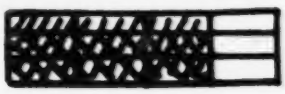
Kaye: Now, I want you to do two-thirds of three-fourths. I want you to start out with the three-fourths and then you would shade it in like this.

Kaye draws the following on the chalkboard:

Kaye: Now, I want you to cut it into thirds and shade in two-thirds of that. So, now we are talking about twelfths. We have six-twelfths and can that be reduced?

The students: Yes, to one-half.

Kaye has on the chalkboard:

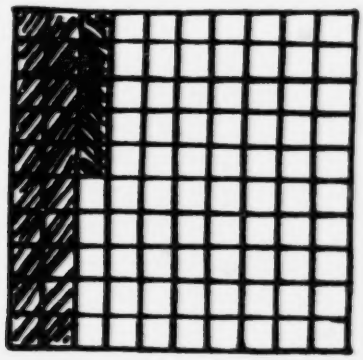


The lesson began with pictures illustrating the concept of halving (already familiar to the students). This was followed by working through problems that showed the concept of multiplication of fractions. Kaye never demonstrated the rules or procedures for multiplying fractions; instead she focused on the conceptual understanding of the algorithm through pictorial representations.

The modifications Kaye made in her fraction unit were carried over to her instruction in the decimal unit. Decimal concepts were introduced by exercises using 100-square grids. The following is an example of a typical start-of-class activity.

Kaye: I want you to draw a graph of the decimal, twenty-five hundredths.

Kaye draws on the board:



Kaye has written the assignment for the students on the chalkboard:

Show on graph paper:	Add
1. .3	6. .6 + .3
2. .03	7. 1.3 + .05
3. $\frac{4}{10}$	8. .15 + .20
4. .40	9. 1.6 + 1.5
5. .7 + .1	10. 2.13 + 3.4

Randy: Do we have to write those as fractions, decimals, and pictures?

Kaye: No, not today you don't.

Kenneth: Thank you, Ms. Kaye.

Jennifer: Ms. Kaye, on problems six to ten, do we have to draw those pictures on for them?

Kaye: No, not today.

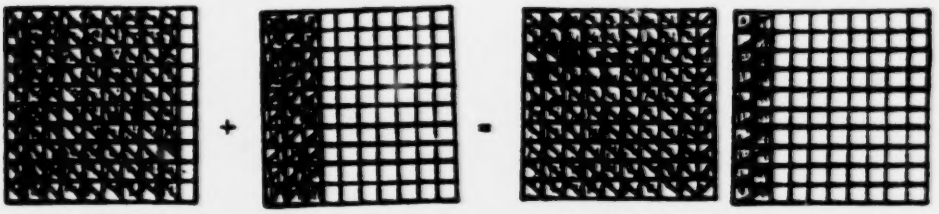
Holly: Do we have to draw those pictures for six to ten?

Kaye: No.

Kaye goes to the board and writes over problems six to ten: NO GRAPHS

At the end of this lesson Kaye described her technique for teaching the students to add decimals to the observer who recorded it.

Kaye told me that yesterday was the first day the students started working with addition of decimals. She had the students draw pictures of the two decimals on 100-square grid paper. She then had them put the two pictures together to come up with the result. She told me she asked the students to represent decimals such as nine-tenths as ninety out of one-hundred squares shaded in. Then she asked the students to look at three-tenths and shade in thirty of a hundred squares.



.90 + .30 = 1.00 and .20

Kaye said she noted when the students combined ninety squares and thirty squares they realized there was one full 100-square grid and part of another one.

She felt this was a very good way to get the students to see that the sum of nine-tenths and three-tenths did *not equal* twelve-hundredths (.9 + .3 = .12). This had always been a common error made by her students.

Kaye said that if the students believed .9 + .3 = .12 before she was now convinced that by using the drawings many of them saw it simply did not make any sense. They were then able to correct their answers to one and two-tenths (1.2).

Kaye thought the same strategies used across the fraction and decimal units helped the students link the two content areas and gain a better understanding of fraction-decimal relationships.

Kaye used the modifications from the fraction and decimal units in her instruction of percents. She used the 100-square grid from the decimal unit to enable students to visualize one hundred percent. She asked students to draw and shade various percent values on the 100-percent grids. In addition, the students were required on most of their assignments to write both the fraction and decimal equivalents for each percent value. She felt these

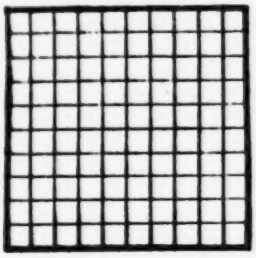
activities helped the students establish conceptual linkages across fractions, decimals, and percents. The following 10-minute activity is an example of the links between the three content areas. At the beginning of class the students were asked to (a) write a definition of percent, (b) to draw a pictorial representation of various percent values, and (c) rename the percents as fractions and decimals.

> The students are entering the classroom and as they enter they take a piece of graph paper and a piece of lined paper from Kaye's desk. They then sit down in their seats and read what she has written on the chalkboard.
>
> Kaye has written on the board:
>
> YOU NEED ONE GRAPH PAPER, ONE LINED PAPER.
>
> Review 5/18/84
>
> 1. Percent means _____
>
> Write as a fraction (simplify completely)
>
> 2. 25% 5. 50% 8. 45%
>
> 3. 100% 6. 33 1/3% 9. 1%
>
> 4. 10% 7. 80%
>
> (10-17) Draw each percent problem (2-9) on 100-square graph paper.
>
> (18-25) Write each as a decimal.

Kaye introduced the students to a 100-percent stick by relating it back to a 100-square grid. She explained to the class that the 100-percent stick was constructed by cutting a 100-square grid into 10 squares each and laying them end to end. The following is a diagram of the 100-percent stick and the 100-square grid:

A 100% stick

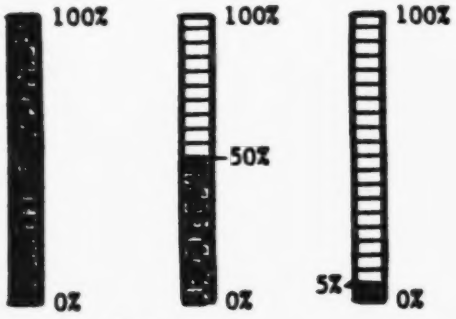
A 100% grid



The 100-percent stick was used to enhance the development of the conceptual understanding of percents. In addition, the 100-percent stick linked the part/whole relationships established in the fraction and decimal units to those in the percent unit. The following observation shows the interaction between Kaye and the students during a lesson in which they were locating percent values on their 100-percent sticks.

Kaye: All right ladies and gentlemen, I wanted you to color in a hundred-percent stick, a fifty-percent stick, and a five-percent stick.

The students had to do the following:



Randy: Would you do a five-tenths-percent stick?

Kaye: How would you do five-tenths-percent?

Holly: That would be just one-half of a square. (Each square in the above diagram has been divided into five equal sections.)

Kaye: Right. All right, do me a twenty-five-percent stick and then follow it with a thirty-three-and-a-third-percent stick.

Some students groan at the request to do a thirty-three-and-a-third-percent stick.

Christine: (Looking at the thirty-three-and-a-third-percent stick) Well, that's just one-third of the stick!

Kaye: Oh my gosh! That's a third of it? So what you are telling me is that thirty-three and a third plus thirty-three and a third plus thirty-three and a third equals ninety-nine and three-thirds?

Well, that can't be right. Your answer would have been a hundred. You only have ninety-nine and three-thirds!

Christine: Yeah, but ninety-nine and three-thirds is the same as a hundred!

Kaye: Right. How many percents are there in two-thirds then?

Kevin and Randy: Sixty-six and two-thirds.

Kaye: Right. Does that mean that twelve and a half percent is one-half of the twenty-five-percent stick?

Randy: Yeah, and it is one-eighth cause it is half of a fourth. Does that also mean that six and one-fourth is one-sixteenth?

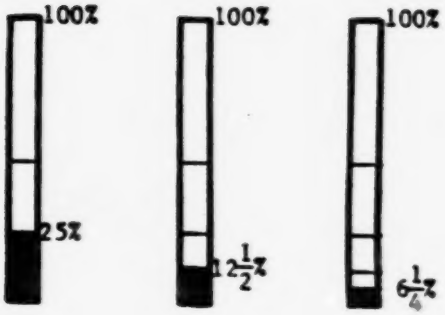
Kaye: Oh my gosh! I'm impressed!

Kaye writes the following on the board:

$$25\% = \frac{1}{4}$$

$$12\frac{1}{2}\% = \frac{1}{8}$$

$$6\frac{1}{4}\% = \frac{1}{16}$$



The interaction between Kaye and the students in the observation above indicated that important linkages had been made between percents, decimals, and fractions. The interaction also indicated that the students were developing a conceptual understanding of these interrelationships. Kaye described her percent unit in an interview.

> The percent unit was just a real total amazement to me this time. Part of the reason is because I did stick with common percents. We did very few things with percents like seventeen percent and four percent, but most of that was conceptual.
>
> I was going to get into problems like sales tax but we ran out of time at the end of the semester, so I never got to that.
>
> I spent two and a half weeks on percents, and two weeks of that was really in terms of the concept of percent, not dealing with moving the decimal back and forth. I did do that one day just to show them that there was another way to approach it.

But I stayed with twenty-five percent and emphasized twenty-five percent, twenty-five hundredths, and a quarter. We just went over that again and again. I feel real good about that. In fact, one of their reviews was that they had to draw four percent, forty percent, and fourteen percent. I gave them 100-square grids to do that with.

The only trouble they had was with the five-tenths percent. I just kept throwing that out there because I wanted them to get that it was half of a percent.

The biggest effect of this teaching was on my role as a teacher. It made it easier to teach percents and decimals once the students had gone all the way through fractions. And for the effect on the students . . . it gave them a better understanding because they had the fraction base.

It all seems so simple now.

Kaye talked about spending most of her time teaching percent concepts instead of teaching the algorithms for computing rates, bases, and percentages (as she had previously done). In addition, she emphasized the importance of the links that were made between percents, decimals, and fractions. Kaye's test at the end of the unit reflected her instructional emphasis on the development of conceptual understandings of percents. She encouraged her students to use the 100-percent sticks, drawings, and any other method they chose to answer the questions on the test which follows:

GENERAL MATH TEST--PERCENTS

SOLVE THESE BELOW:
Show any work on this sheet-Put answers in blanks.

Part I

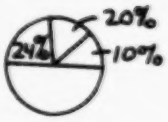
1. 25% of 20 = _____
2. 50% of 80 = _____
3. 33 1/3% of 60 = _____
4. 100% of 90 = _____
5. 200% of 50 = _____
6. 175% of 40 = _____
7. 20% of _____ = 60
8. 25% of _____ = 20
9. 100% of _____ = 8
10. 200% of _____ = 160
11. _____% of 80 = 40
12. _____% of 60 = 15
13. _____% of 50 = 100

PART II Do these circle graphs seem reasonable? Explain why or why not.

1.

2.

3.

4. 

1. _____

2. _____

3. _____

4. _____

PART III Guess what percent of each circle is labeled A, B, C (fill in the %)

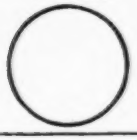
5.

6.

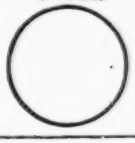
7.

PART IV Draw lines which approximately divide each circle according to the percents given.

8. A=50%
B=40%
C=10%



9. A=25%
B=3%
C=72%



10. A=85%
B=10%
C=5%

The problems in Part I of the test were answered by most of the students using their knowledge of fractional equivalents and the 100-percent sticks. In Part II, the students were asked to explain why the drawings were or were not reasonable. Parts III and IV were problems in which the students had to reverse their thinking; first they estimated percents from given parts of wholes, then they located given percent equivalents. Notably, there were no test questions where the students *computed* answers using algorithms.

At a teacher-researcher meeting Kaye talked about her evolving instruction and how important it was to teach students to be flexible in how they thought about mathematics.

We keep talking about student flexibility, their being able to look at things in different ways.

When I started this project the only way I ever talked about percents was computational, period. So for me to now say, "Oh well, we could look at it this way, or this way, or this way!" That has changed my style of teaching.

I am falling into it more than anything else. Being able to explain it in lots of different ways, I think, would be an example of quality instruction.

As her students became more flexible in their thinking Kaye became more flexible in the way she thought about her instruction.

Units of content that had been previously taught as separate entities were now linked together with common mathematical understandings. The students' new conceptual orientation provided the opportunities for them to realize the mathematical relationships between the content areas which they had not known existed before.

Linkages between new units and previously learned mathematics content. In addition to the creation of linkages across typical content areas, Kaye introduced her students to new units intended to help them develop conceptual understandings of both the new content and previously taught content. The following vignettes describe some of the new topics implemented in Kaye's general mathematics class. Kaye wanted to teach three new units: (a) concepts of and operations with integers, (b) using formulas, and (c) algebraic equations. She introduced the unit on integers through graphing activities. The students were already familiar with plotting positive coordinates from previous activities in a recently completed unit. The following selection describes Kaye's instruction on the first day of the integer unit.

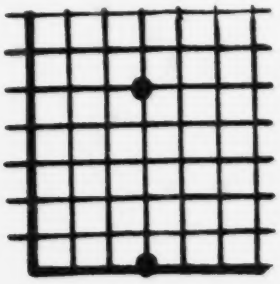
Kaye: (At the board) The pretest on integers you took yesterday indicated to me that you didn't know a whole lot about signed numbers. If you will remember about Morris the Cat [an exercise from the Similarity Unit], what did you do when it told you to find the point 3,0?

John: You went over three and up zero.

Kaye: Right. Now, what would you do for the point 3,5?

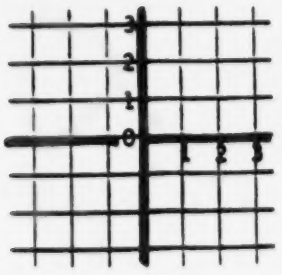
Joe: You would go over three and up five.

Kaye draws a graph and puts in the points.



Kaye: Now, what you have is one portion of what you are going to be doing today. I want you to number the lines--not the spaces--number right on the lines.

Kaye numbers the lines on the graph on the board as the students number their lines on their papers.



Marie: Oh, I remember now! I did this before.

Kaye: Oh no! We can't have this. We can't have you students remembering!

The students continue numbering their graphs as Kaye circulates around checking their work.

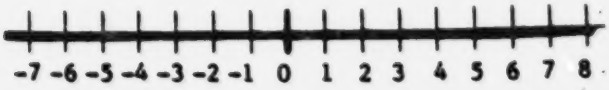
Kaye: All right now, we are going to number all the other lines. These are called axes, by the way. This is the X-axis (pointing to the X-axis on her drawing on the board). This is called the Y-axis (pointing to the Y-axis on her drawing on the board).

If you want to get fancy you can call this the Cartesian Coordinate System. It was named after Decartes.

Kaye writes the name of the system on the chalkboard.

Kaye: I want you to look at the number line I have drawn on the board.

Kaye has drawn a number line next to the graph.



Kaye: I want you to think about picking this up and moving it over here to the graph and laying it down on top of the numbers you have written.

Richard: Oh, so the ones that go down are also minus?

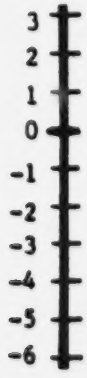
Pete: Are we going to play Battleship (a game using coordinates on a graph)?

Kaye: Oh no! We don't want to have you do something that you already know how to do!

You are all familiar with the thermometer. Now remember when we had those awfully cold winter days and you hoped you wouldn't have school because it was so cold? Well, do you remember what kind of temperature it was when it was that cold?

The students: Below zero, minus.

Kaye draws a thermometer on the board next to the graph.



Kaye: Now I want you to think about moving this line over to your graph and write the negatives in.

(The students write the negatives in on their graphs.)

Kaye: I'm impressed with you guys! So, if tomorrow I put on the board I want you to set up a Cartesian Coordinate System you would be able to do it?

The students: Yes.

Kaye began the lesson with a review of plotting coordinates on a graph. Many students remembered the graphing activities in the last unit and were able to establish linkages that were critical to their understanding of integers. After Kaye was certain the students were following her, she introduced the negative coordinates. Using a number line for the horizontal axis and a thermometer for the vertical axis, she related the numbers on the two axes to concepts that were familiar to the students. The above segment was followed by a controlled practice activity in which the students worked along with her in plotting coordinates and connecting them with lines to make a picture of a car. The links Kaye made to previously learned content and familiar concepts helped the students begin to understand the new concept of integers. Kaye talked about the use of graphing as a vehicle for introducing integers in the following interview.

> I think that the graphing experience is helpful when you get into the algebra area. I use it as a tool for working with the integers, which takes the students into algebra.
>
> Graphing itself--I might throw that out. But I like it just because it gives students a different way of looking at numbers.
>
> One of my goals for the algebra unit was that they end up not being afraid of it. I don't think there was anybody that felt, "Oh, Goll, I could never do algebra!"

Kaye used graphing to introduce her students to integers. After the unit on integers had been completed she started a unit on formulas. The unit on formulas was used to provide students with different opportunities to review fractions, whole numbers, integers, and decimals. The following example is from part of the final test Kaye gave to her students on formulas.

General Math-Algebra Introduction

Evaluate the given expression using X = 5 and Y = 3

1. $X + Y$
2. $X - Y$
3. $Y - X$
4. XY
5. $\frac{X}{Y}$
6. $\frac{Y}{X}$
7. $3X - 2Y$
8. Y^2
9. $X^2 + Y^2$
10. X^3
11. $16^{(Y)}$
12. $\frac{X + Y}{2}$
13. XY^2

EXTRA CREDIT: Evaluate using $X = \frac{3}{5}$ and $Y = \frac{1}{2}$

A. X^2Y B. $X + Y$ C. XY D. $Y-X$ E. $\frac{X}{Y}$

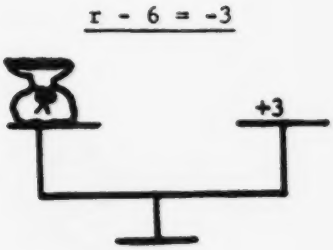
(These were used with fractions, decimals, integers as a review at the end of the year.)

Although the values of the variables on the test were whole numbers, Kaye used other variables including fractions, decimals, and integers. This provided a review of basic computational skills in a new context. The unit on formulas provided the opportunity to review previous content and helped develop conceptual links across the topics of integers and algebraic equations.

Kaye's third unit, algebraic equations, followed her units on integers and formulas. The focus on this unit was on the development of conceptual understandings of equations. The following selection shows how she taught her general math students to think about solving algebraic equations.

Kaye: There are two ways of approaching problem twelve. I can think of subtracting a six. Here is my sack with an unknown r in it. If I take six out, then what do I have to do to balance my scale again?

Kaye has drawn the following on the board:



The students respond: You have to add six.

Kaye: The problem, r minus 6 equals a negative three, is about the hardest kind of problem to do. If I had *X* minus a negative four equals negative two, then I would have to take 4 out of my sack, but then I would have to put 4 back in so I would have both sides balanced. It is a very hard problem. It is very hard to logic it out.

Kaye draws the following on the chalkboard:

Kaye: If I give you 7 *X* plus 3 equals 38, what are you going to do to solve for *X*?

Randy: Boy! I'm really lost! Is the *X* an *X* or does it mean to multiply?

Kaye: That's a good question. One of the things that we have done all along is that when we start algebra problems we don't use an *X*, we use a ...

Randy: Oh, I remember now, a dot.

Kaye: You are right.

Randy: I forgot about the dot when I asked the question.

Kaye:	I took seven *X*s all together and I added three extra chips and I kept adding chips on the other side till I got 38 and it balanced my scale. You would take away three from both sides and what do you have left?
The students:	Seven *X*.
Kaye:	What's on the right?
The students:	35.
Kaye:	So, Miss Dwyer, what's the answer?
Mary:	5.
Kaye:	So, if we put it back into the equation then we would have 7 times 5 plus 3 which is 38.
Randy:	I know these now! I must be getting smarter!
Kaye:	I'll tell you something, you have done in two days what it usually takes me two weeks to do with my algebra students!

In the description above Kaye illustrated the concept of equations by using a balance scale. The solution to an equation was demonstrated through the use of models and pictures. She used controlled practice, pictorial representations, concrete explanations, and questioning to help students understand the content of this new unit. Kaye reported her reactions to this unit at a teacher-researcher meeting:

> In my math classes I have been attempting to teach integers using graphing which we've done before with operations with integers. It's been working out real well. Last couple of days we have been doing some preliminary algebra to the point where today we were working on things like $3X + 2 = 7$ and how would you solve that. I am really pleased with the results.
>
> Most of them have an idea what is going on and the operations of integers has gone really well. I've not done that with general math kids except at the very end when you teach them a negative times a negative is always positive. I have never gotten very far with it. It is going very well. I am really pleased.

Kaye's satisfaction with the success of the unit seemed to be the result of the techniques she used and the linkages the students had made across the various content areas.

Kaye taught a new unit on problem solving to help her students develop their ability to solve problems and to provide them with another way to review basic computational skills. During this unit, she continually asked the students to explain their thinking about how they solved the problems. An example of one such problem and its solution is seen in the following observation.

During one period the students were asked to solve the following problem:

> It takes 12 minutes to cut a log into 3 pieces. How long does it take to cut a log into 4 pieces?

After the students were given a few minutes to work on the problem, Kaye initiated the following discussion.

Kaye: All right, let's try this third problem. "It takes 12 minutes to cut a log into 3 pieces. How long does it take to cut a log into 4 pieces?" Who has the answer?

Jeff: 16 minutes.

Kaye: Is making a model going to help you solve this problem?

Alice, Mary, Jeff: No.

Alice: I divided 12 by 4 to get a quotient of 3. Then I added 12 and 3 to get 15.

Alice then changed her answer to 12 divided by 3 to get 4. She added 12 to 4 and got 16. Alice assumed she had to make 3 cuts in the log rather than only 2 cuts.

Mary: Well, it depends on what you're cutting with. It depends on how big across the log is.

Sue: Do we make 3 cuts or do we make 3 pieces?

Kaye: Sue just raised a question. Do we make 3 cuts or do we have 3 pieces?

This question tends to clarify many of the students thinking about the answer. Many students look at their answers and change them because they realize that only 2 cuts are needed to make 3 pieces.

Don: Well, I got the answer that 3 goes into 12, 4 times. So, 4 is the answer.

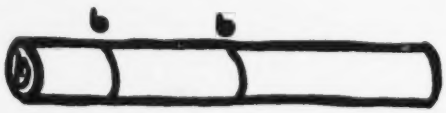
Jeff looks at Don from the other side of the room and is frustrated that Don still can't seem to realize that only 2 cuts are needed, not 3. Jeff gets up from his chair, walks over to Don's desk and takes Don's pencil from him and starts drawing a log on Don's paper.

Jeff: Look! I am going to have to do this for you! I'll draw a picture for you!

Jeff draws the picture of a log with 2 cuts in it resulting in 3 pieces.

Kaye: Jeff, show us up here what you did.

Jeff goes to the chalkboard and draws a log. He makes 2 cuts in it and above each cut he writes a 6.



He then makes a third cut in the log and writes a 6 over that to show the class that there are 3 cuts taking 6 minutes each.

Kaye: What did these guys do to solve the problem?

(She is asking the class to tell her which problem-solving strategy they used to solve the problem.)

Don: Guess and check.

Kaye frowns at Don's wrong response.

Don: I found a pattern.

Kaye still frowns at this wrong answer.

Kaye: Class, what did Jeff do to solve the problem?

The class
(and Don): He drew a picture.

The students were actively involved and interested in the discussion of the solution to the problem. Those who had an incorrect answer wanted to understand why they had made their mistake. Once Sue clarified the problem by her question, the students were able to rethink their results and many corrected their errors. Don (who remained confused) persisted in the discussion of how the answer was obtained. Jeff (knowing the answer) wanted to go on to the next problem, so he helped Don by drawing a picture for him. Although the students cared about having the correct answers, they seemed more concerned with understanding the problem and its solution. They no longer simply accepted an answer as either right or wrong, they discussed why the answers were right or wrong. In an interview after the problem-solving unit Kaye was asked about the factors she thought motivated students to learn mathematical content.

Nason: What motivates your students to learn the math content?

Kaye: There's a certain amount of motivation to getting their work done when they first come in here.

As time goes on, I think there is some intrinsic interest in the problems themselves that serves as a motivator.

There is also the response of just pursuing a problem for the sake of the problem. If it was kind of an interesting problem so the students would say, "Let's do it just to find out what we're going to come up with."

Kaye was also asked about the importance of having students develop skills in problem solving. Her response emphasized the value of developing problem solving abilities as a final mathematical goal and the importance of computational skills as the tools used to reach that goal.

Nason: How important do you think it is for students to develop skills in problem solving?

Kaye: It all comes back to computation. We as a whole society push computational skills every time we test students. Even the *Nation At Risk* book did that. Those things are always highlighted.

With problem solving, I think if you can solve a problem in a certain context then you can apply computational skills to figuring out the result. For example, figuring out how much asphalt sealer you'll need for your driveway.

Problem solving is the final goal we're trying to reach. We spend an awful lot of time with computational skills because if you can't compute you're going to have problems with the problem solving.

Problem solving is, I guess, the final goal. If I can get through the other things then that's what I'm trying to get to in the end.

Among the other new mathematical units Kaye added to her general mathematics curriculum was a unit on probability. Kaye and her students spent about 4 weeks working through the probability activities included in the Probability Unit developed by the staff of the Middle Grades Mathematics Project.[7] Kaye's feeling about using new content, such as this unit, was that it gave the students another way to see interrelationships between various mathematics topics and to give them additional computational practice. In an interview, Kaye talked about the value of the Probability Unit.

Nason: How important was the Probability Unit to your students learning mathematics?

Kaye: The probability unit is not something that I would have said would have been important to teach all by itself. My students will survive quite nicely if they never have any probability.

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[7]This material is referenced at the end of the case study.

> Why I see the probability unit as being worthwhile is that it is a method to get at those computational skills. So, what I guess I'm still saying is that, in the end, the computational skills is where we're headed.
>
> If I had a student who knew everything else in my class but couldn't do probability, I wouldn't feel real terrible about it. I do think that the probability unit gives me a nice vehicle for teaching the other things like fractions and problem solving.

The introduction of new topics into the general mathematics curriculum afforded the opportunity for students to experience interesting, challenging, and unfamiliar mathematical content. While the students worked on the activities of the new units Kaye continuously engaged them in dialogues which enabled them to understand the linkages and interrelationships that existed between these new units and the mathematics with which they were already familiar.

Summary. The mathematical content of Kaye's computationally oriented class had been transformed. The daily seatwork tasks of numerous and mundane problems that simply required repeated algorithmic applications evolved into interesting and challenging assignments involving a variety of activities and responses. The content of general mathematics evolved from the continuous sets of computational reviews of whole numbers, fractions, and decimals into a series of related ideas linked together by common mathematical concepts. Although the ability to compute with accuracy remained a valued objective for Kaye, it too, had evolved from being a goal into a skill that gave students the tools they could use to help understand and solve mathematical problems. She continued to work to improve the content throughout the duration of the project. Kaye reflected on the outcomes of some of her own improvements and her plans to continue to make changes.

Nason: Can you think of any improvements you could make that would improve general mathematics?

Kaye: I will probably approach it in much the same manner as this year. I feel pretty good about what went on this year.

I definitely like the Probability Unit, although I would try and shorten it up. It may mean removing some material, or it may mean just moving faster through that material.

I think that I would probably not alter the percent unit. I would like to make it more workable.

I would definitely do the fractions unit again, working with the hands-on types of things. I will probably take a strong look at tying fractions, decimals, and percents together.

I will do the estimation unit again. I think that's important. It goes along with problem solving and needs to be done intermittently throughout the year.

I am comfortable with doing units in blocks of time, but I also think there's the need to tie it all together. That's something I have not done in the past.

Kaye was beginning to realize the importance of linking the content and tasks together as one way to improve both the learning and instruction of general mathematics. In addition, she identified changes she would have to make in some units when she taught them again. The modified units and new units provided her with the opportunity to teach mathematics in a variety of ways she had not done before. Through the implementation of new and modified content Kaye believed she had become a better teacher, one more capable of using a variety of different ways to think about and teach mathematics. The following interview segment captures Kaye's perception of herself as a general mathematics teacher at the end of the project.

Nason: What is your perception of yourself as a general mathematics teacher now?

Kaye: Much improved, in all kinds of areas. I think probably the biggest thing I've been more aware of has been that I have become more flexible in my thinking in terms of teaching. I have always tried to find different ways of approaching the same content. I know kids don't learn in just one way, but I've always felt like I didn't have a lot of resources. I feel much more comfortable with that now.

I think the percent unit was when it really hit me. Not only were my students getting more flexible in their thinking, but I was getting a lot more flexible in my methods of presentation. That was enabling me to be a much better teacher.

The evolution of mathematical content occurred when Kaye became more flexible in her thinking about learning and instruction. She realized the importance of establishing conceptual links within each mathematical topic (i.e., fractions) as well as across mathematical topics (i.e., fractions-decimals-percents). She also became aware of the importance of new content and topics to help her students realize the linkages or interrelationships which existed in mathematics.

The Evolution of Communication About the Mathematical Content

Questioning the responses of students in whole-group instruction has been one of the biggest changes I have made. I think it's probably the most positive thing I've done.

Questioning has been helpful to me because it helps me find out where student's misconceptions are. It helps the students realize that they aren't alone with their misconceptions and wrong answers.

Questioning is important because it helps them think about why they came up with what answers they got and also it helps them see how other people solve problems.

(Pamela Kaye)

Communication about mathematics in Kaye's computationally oriented class had been limited generally to giving directions for assignments and reciting answers. These communication patterns were quite suitable for the instructional goals and objectives of a computational class in which students were

required to simply follow directions and accurately compute answers to a set of problems. Since the content and tasks had been little more than computational reviews, only minimal mathematical communication needed to take place. When Kaye began changing the orientation of learning and instruction from computational to conceptual, the strategic instructional task of content communication had to be modified to reflect this new focus. A conceptually oriented class requires communication patterns different from those of the computationally oriented class--questioning, discussing, and explaining needed to emerge from the existing communication patterns of telling and reciting. Mathematical dialogues during whole-group instruction needed to evolve from the off-task socializing during seatwork that had prevailed in the computation oriented class. Mathematical language that encouraged conceptual thinking needed to evolve from the nonmathematical language they had been using. Sizer (1984) discussed the value of questioning in promoting the development of student thinking:

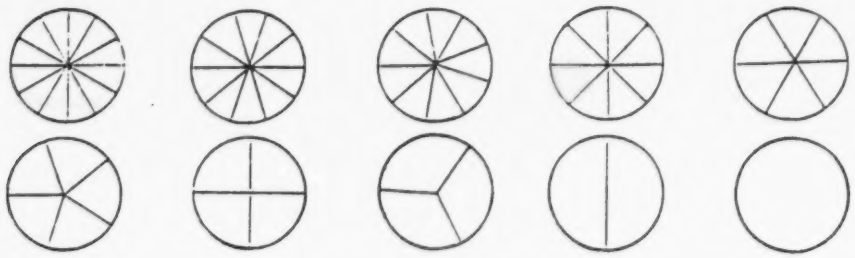
> Schools that always insist on the right answer, with no concern as to how a student reaches it, smother the students' efforts to become an effective intuitive thinker. A person who is groping to understand, and is on a fruitful but somewhat misdirected track, needs to learn how to redirect his thoughts and to try a parallel but somewhat different scheme. Simply telling that person that he is wrong throws away the opportunity to engage him in questions about his logic and approach. (p. 105)

If Kaye expected her students to develop conceptual understandings of mathematical content, then they would have to acquire a new, richer mathematical language. She would have to modify the strategic instructional task to provide ways to enable the students to express their thoughts, ideas, and questions with clarity and precision as they participated in mathematical discussions.

This section includes descriptions of the modifications of this strategic instructional task which Kaye used to develop a mathematical language to help

her students create linkages between the related math concepts. Also, this section includes Kaye's illustrations of the techniques she used in the several content areas to enhance mathematical communication.

Establishing content-concept linkages. The evolution of mathematical communication required the creation of a mathematical language and mathematical experiences that focused on conceptual development. Kaye used activities with manipulable materials as an instructional method to provide students with a set of commonly shared experiences from which they could begin to build a conceptually oriented mathematical language. The student-made fraction kits used throughout the fraction unit is an example of this method. Each student made a fraction kit by coloring and cutting 10 fraction circles into the following parts.



The students stored these fraction kits in their math folders and used them for various activities throughout the unit. The following selection describes an activity in the fraction unit in which the fraction kits were used to solve a problem.

Kaye: Okay ladies and gentlemen, could you look up here for a moment. When you finish the review take your fraction kit pieces out and take one of the halves and see if you can make a half by combining two other colors. Using different colors, see if you can make a half. See how many different combinations you can get.

Kaye writes on the board:

Combine: $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

The students start working on their assignment.

(Later) Kaye walks over to Russ and Ron and tells them to go to the board and start putting their answers up. Soon after instructions are given Kaye tells other students to write their answers if they have different answers than the ones that are already up. The students have the following answers on the board:

Russ

$\frac{1}{6} + \frac{1}{8} + \frac{1}{12} + \frac{1}{12} = \frac{1}{2}$

Jim

$\frac{1}{12} + \frac{1}{6} + \frac{1}{8} + \frac{1}{6} = \frac{1}{2}$

Dick

$\frac{1}{3} + \frac{1}{8} + \frac{1}{12} = \frac{1}{2}$

Stan

$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{2}$

Sue

$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} = \frac{1}{2}$

Marie

$\frac{1}{3} + \frac{1}{3} + \frac{1}{2} = \frac{1}{2}$

Mike

$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$

Sandy

$\frac{1}{3} + \frac{1}{8} + \frac{1}{12} = \frac{1}{2}$

Although only Stan and Mike were correct, Russ, Jim, Dick, and Sandy were nearly correct, in that they were only off by $\pm$ 1/24. Sue's answer was off by 1/12, and Marie's answer was closer to 1 than to one-half. The fraction pieces allowed the students to become actively engaged in thinking, talking, and working with fraction concepts. The activities with the fraction pieces gave students the opportunity to interact with one another and to begin to communicate about concepts they were studying.

The fraction kits also provided the students with different ways to think and talk about the various concepts within the unit. In the following observation, Kaye used the fraction pieces to help the students develop a conceptual understanding of fractional inequalities.

Kaye: I want you to think about three-fifths. I would like you to get your pieces out. Is three-fifths greater or less than or equal to one whole?

The students take their fractional pieces from their envelopes and lay the pieces on their whole circles. They see the three-fifths is less than one whole.

Kaye writes on the board the following:

$\frac{3}{5}$ is less than

$\frac{5}{5}$ is equal to

$\frac{6}{5}$ is greater than

Kaye: Does anyone remember the sign for *less than*?

Karla: An alligator.

Kenneth: Mrs. Jones showed me.

Randy: It looks like Pac Man.

Kaye: My algebra students forget this all the time.

Karla: That's surprising, algebra students are supposed to know everything! I know the big part goes to the biggest fraction, because you want the biggest part of the pie.

Kaye: Which one of these signs, the A sign or the B sign, would you use with three-fifths?

Kaye writes on the board the following:

A $<$ B $>$

Kaye: Then three-fifths is less than 1 and so you would write it like this.

Ms. Kaye writes $\frac{3}{5} < 1$

Karla: Those were on the assessment test. You know, those problems like, "which is bigger?"

Kaye: Sometimes those are hard.

Karla: Yeah, sometimes.

Randy: Yeah, but I understand them now.

Kaye: What about three-fifths, three-eighths, and three-twelfths?

Karla takes her fraction pieces and lays the three-fifths, three-eighths, and three-twelfths over each other so she can compare them.

Karla: The three-fifths is bigger than three-eighths, and the three-eighths are bigger than the three-twelfths.

Kaye draws pictures of the fractions on the board.

$\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{12}$

Kaye: All right, I have one thing for you to do today. Notice that the directions tell you to use your pieces to check if you need to.

Kaye writes an example on the board.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

The students start working.

Later, after the bell rings and the students are dismissed Randy talks to Kaye.

Randy: This is the first time I ever understood fractions! It's the first time I ever could do them!

Kaye: When did you start having trouble with fractions?

Randy: The first time I didn't have trouble with math was in kindergarten.

The observation illustrated how Kaye and the students talked about the concept of fractional inequalities. Questioning, explaining, and discussing of these concepts during whole-class instruction occurred more now than ever before. The strategic instructional task of communication about the content had been modified. In the observation above Kaye did not demonstrate the computational methods for determining inequality or equality relationships such as (a) the calculation of cross products or (b) the changing of each fraction to an equivalent fraction with the same denominator then comparing the numerators. She encouraged the students to think and talk about the concepts of fractional inequalities.

In a later lesson on fractional equalities, Kaye and the students engaged in a discussion. She asked them to explain their thinking, provide a description of this concept, and use precise mathematical language.

Kaye: What does one-half equal in fourths?

Randy: Two-fourths.

Kaye: What can you do?

Lenore: Yeah, you double it...never mind...

Mary: Yeah, you draw another diameter, and then you shade it in.

Kaye draws the following:

Kaye: All right, how would you show it with arithmetic?

Mary: Two times two is four and two times one is two.

Kaye draws the following on the board:

Kaye: Is this halves?

The students: Yes.

Kaye: How do you divide the circles up into sixths? Jessica, do you know?

Jessica shows Kaye from her seat by pointing to her paper.

Kaye: I can't see it from here.

Jessica: Well, you go like that . . . draw one down the middle.

Kaye follows Jessica's directions and draws the following on the board.

The students: No.

Kaye: Why?

The students: They have to be congruent pieces.

Kaye: All right.

Mary: Make the one-half into thirds and shade the top three.

Kaye: All right Jim, can you show me how you would do the arithmetic?

Jim: You would shade it in.

Randy: Make it into eighths.

Kaye draws on the board:

Kaye: Okay, with one-half with pictures now, how many eighths would that be?

The students: Four.

Kaye: All right, the numerator tells you...,

The students: How many pieces you have.

Kaye: The denominator tells you....

The students: How many pieces there are in the whole thing.

The selection illustrated the evolution of the mathematical communication across the fraction unit. The class moved from an emphasis on manipulable materials to pictorial representations as they discussed and described fractional concepts. Many students frequently used their fraction kit pieces to check their answers to problems, which were obtained from drawing pictures or calculating. Their experiences with manipulable materials and pictorial representations in class activities and discussions provided a common mathematical language that helped them develop a conceptual understanding of fraction relationships. Kaye's emphasis on using mathematically precise language while explaining or describing a concept or answering a question helped her students become familiar with a language that would enhance their understanding of fraction concepts. By the end of the fraction unit the students were more articulate in describing and discussing their thoughts and ideas about mathematical concepts.

Creating linkages across units. Kaye used modifications of the strategic instructional task of content communication to make the students aware of the commonalities that existed across different areas of content. Examples of these modifications included finding error patterns, discussing multiple representations, and providing feedback on student math improvement. She used common mathematical errors to initiate class discussions. During these

discussions she and the students would talk about the probable kinds of erroneous thinking that resulted in the mistakes made. In the following observation, Kaye engaged the students in a discussion of error patterns as she tried to get them to think about equivalent fractions.

Kaye: When you are showing your answer for number one you need three circles and your pictures should look like this.

Kaye draws on the board:

$\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$

Kaye: When you show *not equivalent*, your picture then will have fractions but they won't be equal. How many eighths are equal to one-fourth?

The students: Two.

Kaye: Jessica, can you tell me why this one girl in my sixth hour said that two-eighths was not equal to three-twelfths?

Jessica: No.

Kaye: Amy, what do you think?

Amy: Well, they might think that 8 won't go into 12 evenly so the two fractions aren't even.

Kaye: Right.

Amy: Yeah, that's what I did.

Kaye: All right, I want you to go back and I want you to look at your pictures. Just because one number won't divide evenly into another it doesn't mean they're not equivalent. Is one-fourth bigger than one-sixth?

The students: Yes.

Kaye: But the 6 is the bigger number.

William: But it is a smaller portion of the whole!

Kaye used an error made by a student in another period as a way to have her students think and talk about equivalent fractions. Amy answered Kaye's question correctly because she had made the same mistake. The technique of talking about common errors continued into the unit on decimals. In the following observation Kaye's students discussed errors that had been made on a quiz. Kaye gave her students a quiz on addition and subtraction of mixed numbers, and then collected their papers. She wrote some of the wrong answers on the chalkboard and started a discussion of these errors.

Kaye: I want you to look at these and I want you to do a kind of error analysis so you can see what you did.

Kaye refers to the first problem she has written on the chalkboard:

$$\begin{array}{r} 12 \\ -\ 1\frac{3}{8} \\ \hline 11\frac{3}{8} \end{array}$$

Mike: They probably went--12 take away one and three-eighths leaves eleven and three-eighths. Well, you can't take three-eighths from nothing!

Kaye: Right, you know that you have to borrow.

Kaye refers to the second problem written on the board:

$$\begin{array}{r} \frac{96}{8} \\ -\ 1\frac{3}{8} \\ \hline 1\frac{93}{8} = 11\frac{5}{8} \end{array}$$

Mary: Ninety-six-eighths take away one and three-eighths leaves one and ninety-three-eighths which is eleven and five-eighths.

Kaye: Yes, well you see, they can't leave the 1 because you are going to have to subtract the one and three-eighths *not just* three-eighths. (Kaye refers to the next problem written on the board.)

$$\begin{array}{rr} 1\frac{1}{2} = & \frac{10}{10} \\ - & \\ \frac{3}{5} = & \frac{6}{10} \\ \hline & \frac{4}{10} = \frac{2}{5} \end{array}$$

Kaye: How about one and one-half take away three-fifths? I was surprised to find that three people got this wrong.

Mike: Well, they did the one over, but they forgot to do the one-half.

Kaye: Right, they changed the 1 to ten-tenths, but then they forgot to change the one-half into five-tenths. We should have had fifteen-tenths. They got everything right but that one little thing, that one little procedure wrong. (Kaye writes the following problem on the board.)

$$\begin{array}{rr} 12 & \frac{60}{5} \\ - & \\ \frac{3}{5} & \frac{3}{5} \\ \hline & \frac{57}{5} = 5\overline{)57}^{\,11\ R2} = 11\frac{2}{5} \end{array}$$

Kaye: All right, now Beverly.

Beverly: Well, put the 12 on top and the three-fifths under that and then you would multiply 12 by 5 to get 60 over 5, and 5 times 1 is 5 and 3 times 1 is 3, and 60 over 5 take away 3 over 5 is fifty-seven-fifths.

Then 5 into fifty-seven goes 11 times and you have 2 left over, so the answer is eleven and two-fifths.

Kenneth: Gee, you can do it that way?

Kaye: Yes, you can. Is there another way?

Mary: You could take the 12 and make it eleven and five-fifths and then you could take away three-fifths which would leave eleven and two-fifths.

(Kaye writes the problem on the board as Mary talks.)

$$\begin{array}{rcl} 12 & = & 11\frac{5}{5} \\ -\ \frac{3}{5} & = & \frac{3}{5} \\ \hline & & 11\frac{2}{5} \end{array}$$

(Kaye writes the next problem on the board.)

$$\begin{array}{r} 6\frac{1}{2} \\ -\ 3\frac{7}{8} \\ \hline \end{array}$$

Kaye: All right, Karla.

Karla: All right, six and one-half take away three and seven-eighths.

(As Karla talks through the problem Kaye writes her answer on the chalkboard.)

$$\begin{array}{rcccl} 6\frac{1}{2} & = & \cancel{5}\ \frac{4}{8} & = & \frac{9}{8} \\ -\ 3\frac{7}{8} & = & 3\frac{7}{8} & = & \frac{7}{8} \\ \hline & & 2 & & \frac{2}{8} = 2\frac{1}{4} \end{array}$$

Karla: You change the one-half into four-eighths and three and seven-eighths stays seven-eighths. Then you do something with the 6 to make it a 5, and that gives you nine-eighths. Take away seven eighths and nine-eighths and that is two-eighths. 3 from 5 is 2. So your answer is two and two-eighths or two and one-fourth.

Kenneth: I think she did it wrong! I think she was supposed to add four-eighths to eight-eighths and get twelve-eighths. Then I think she was supposed to take seven-eighths away from that.

The students were asked to give explanations for the errors and to give several different ways to explain how to find the correct solutions. If a student made a mistake while explaining a solution, Kaye would wait until another student noticed it and caught it before she would point it out (as Kenneth did with Karla's mistake). Error analysis remained a technique she implemented throughout the year. This enabled the students to realize that many of their classmates made the same mistakes and gave them the opportunity to rethink problems from the perspective of an analyzer rather than the solver of the problems.

In addition to the technique of error analysis as a way to increase communication, Kaye encouraged her students to think about problems in a variety of ways that included using manipulable materials, drawing pictures, and computing. In the following selection Kaye asked the students to combine three fractions and to tell her how they arrived at their answer.

Kaye: Suppose I wanted you to combine one-half plus one-third plus one-sixth?

Kenneth: Oh, I know, I know!

Kaye: I want you to tell me how much.

Kenneth: Three-twelfths. . . no, no, that's not right. Wait, I can redeem myself.

(Kaye has the following problem on the board:)

$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{3} \\ +\ \frac{1}{6} \\ \hline \end{array}$$

Kaye: All right, I'll give you a chance to redeem yourself.

Kenneth: It's three-elevenths . . . or . . . one whole.

Kaye: Kenneth, we just said over here that you can't add numerators and denominators!

Kenneth: Well then, how can you add that?

Randy: Because it's one-third plus one-sixth which equals one-half, and one-half and one-half equals one whole!

Melanie: I did it differently. I put one-half, one-third, and one-sixth in a line and I got a common denominator and I added them.

(Kaye writes on the board what Melanie has told her.)

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{1}{6} = \frac{1}{6}$$

Kaye: All right, so you did the arithmetic way to get your answer. You are correct, but can anyone do it with a picture?

Randy: I can.

(Randy goes to the board and draws the following.)

Randy: Here is the one-half, and here is the one-third, and here is the one-sixth. You can see they are equal to one whole.

Kaye: Yes, you could do it that way also. I don't care if you do it this way with the arithmetic or not. I do care that you do it with a mental picture. Because if you can draw a picture you may avoid adding across. If you added across you would get three-elevenths.

(Kaye shows the students the following on the board.)

(Randy's)

(Kenneth's)

Kaye: You see, if we draw a hexagon like Randy's and divide it into twelfths we can see that elevenths is really close to twelfths. If we took this hexagon and divided it into twelfths and shaded in three of those this is about how much three-elevenths would be. Does that even look close to the answer Randy gave us, one whole?

The students: No.

Kaye let Kenneth live with his mistake until Randy and Melanie finished their explanations for how they arrived at the answer. When they were done, Kaye returned to Kenneth's answer and compared it to Randy's. This showed the students how Kenneth's inaccurate algorithm provided him with an answer that was not reasonable after it had been pictorially represented.

Another technique used to modify the instructional task of communication was that of giving the students more feedback on their overall mathematical achievement. Kaye thought this would be one way to encourage the students to continue improving their mathematical communication. When she showed the students the progress they had made across a semester or a unit (using pretest-posttest gains), she emphasized that their achievement was largely due to their participation and class communication. She talked about the value of using pretest-posttest feedback in the following interview.

> The effect of using the pretest and posttest gains with the students was really interesting. When they could see that there was a measured improvement in their scores, their eyes got big. When I told them that I expected only about a four-tenths of a year gain from September to December and they did much better than that then they were really surprised.

Even my worst LD student went up eight-tenths instead of only four-tenths. He started out in the basement, the sub-basement. So, although he knows he's not really good at math, he could see that he had made some improvement. If I hadn't done the pretest-posttest thing I wouldn't have had anything to give him. I could have told him that, but I really don't think he would have believed me. But there was a number there that told him, "Yep, you did it!"

I think I could see a real difference for the majority of students. They know they improved. I really think that has made a big difference in the fact that they are finally interested in learning some math. Although they know it's not their favorite thing to do they are telling me, "O.K. Tell me more!"

I think I perceive an attitude change. Maybe it's not there, but I perceive an attitude change.

Kaye believed the feedback convinced her students that they had learned and were, in fact, capable of being successful in mathematics. Kaye reflected on the effect of feedback she gave to her students regarding pretest-posttest gains in the following interview.

Nason: What factors are motivating your general math students in class?

Kaye: I think this year it was self-improvement. Particularly after the first semester. I think part of the reason for them was that as I went over the total class improvement from the beginning of the semester to the end they saw some of the people next to them who they thought were just as dumb as they were and who had improved a grade level or more in one semester's time. They decided there might be some hope for them yet.

Nason: Would you comment on the attitudes of the general math students towards learning?

Kaye: I think that the Shaw-Hiehle test was really helpful for them in changing their attitude toward learning. For them I think it was a pretty painless year.

Nason: What about their attitudes towards achievement?

Kaye: The thing that comes to mind was I hope they got out of this an attitude that said, "I'm O.K." When I could hand back their test results and tell them they had improved two or three grade levels and they were now at the ninth grade level in mathematics, well, they were kind of proud! They would say, "I'm not the smartest kid there is around, but I know I can do it. I did it!"

> That's the way I felt they thought about their achievement. I think at the beginning of the year their attitude toward achievement was that they couldn't be successful. They really thought they were dumb.

Kaye believed student interest in mathematics was improved as a result of the feedback they received from her. While pretest-posttest gains will be discussed in a later section, an example of this feedback will be presented here. The pretest-posttest scores and grade level equivalents of three students (identified in previous vignettes) on a computational test are included below.

Table 1

Pretest and Posttest Scores on the Shaw-Hiehle Computation Test[a]

	Pretest (September)		Posttest (June)	
	Score	Grade Equivalent	Score	Grade Equivalent
Randy	15	4.4	39	8.3
Kenneth	15	4.4	34	7.5
Karla	26	6.6	52	10.2

[a]The Shaw-Hiehle Computational Test includes 60 items in which the students must calculate the answers. There are 20 items involving whole number operations and 10 items each with fractions, decimals, percents, and applications.

The feedback Kaye gave to the students included discussing their improvement of mathematical skills across each semester and the year and emphasizing the importance of participation and communication to the improvement in their scores.

Another method Kaye used to increase the students' mathematical communication was that of questioning. She believed that asking students questions throughout the year would increase the quality and quantity of responses which would lead to mathematical improvement. Her objectives for continually questioning the students were (a) to encourage students to *become aware of and to describe their thinking* as they solved problems, (b) to have students *use mathematical language* when they explained solutions to problems or when they asked questions, (c) to give students the chance *to learn about the different ways their classmates thought about and solved the same mathematical problems*, and (d) to help *diagnose where the students were* in terms of their mathematical understanding of the content they were studying.

On the first day of the second semester, Kaye explained to the new students in her class why she asked so many questions and why she expected them to also ask questions.

> Asking questions is important. I ask you questions all the time because I need to know how your mind works. And, besides, it usually helps others to understand.
>
> I want you to ask a lot of questions in here.

The students who had been in the class since September were familiar with Kaye's continual questions. They were also used to her asking them to explain, discuss, describe, and defend their answers. The new second semester students did not have the same opportunity. At a teacher-researcher meeting, Kaye recalled the first day of the second semester and the differences she noticed between the students who had been with her since September and those who were new to the class. She also talked about the value of questioning as a technique for increasing the mathematical communication in the class.

> You keep asking a question over and over again and it finally gets to a point where the students know what it is you are looking for. For example, when you say to them, "You have four-sixths and that equals two-thirds--*why* does it equal two-thirds?" Eventually the

> students will begin saying, "*Because* you can divide the numerator and denominator by two." They start answering in complete sentences. They are not giving you just the yes/no-type answers anymore because they know I will ask them, "How did you get that?"
>
> I think we are forcing some of that explaining to happen with questioning.
>
> I noticed that particularly when the semester changed. I had 10 new students. I reviewed fractions with them, and when we went over the questions, the students I had the first semester answered the questions differently than did the new ones.
>
> The students who were new just gave an answer. The students who were not new gave an answer and an explanation. I think I am asking them to do more of that all the time and I think that is really important.

Although questioning the students was Kaye's primary communication technique implemented throughout the year on a daily basis, there was also an emphasis on using a mathematical vocabulary. She used mathematical vocabulary during whole-class dialogues and discussion, and in the seatwork period. Any questions, descriptions, and explanations included the use of mathematical language. Kaye reflected on the technique of using precise mathematical vocabulary at a teacher-researcher meeting:

> I am stressing vocabulary much more than I ever have--that came naturally in my Geometry Unit. I am concerned that they understand the terms. I think when I did the Problem Solving Unit, one of the biggest problems the students had was they didn't know the vocabulary. So we spent a lot of time talking about the terms.
>
> I would have never worried whether the students knew what "isosceles" meant or not--and I am still not really worried about that, but I do want them to understand that word so when they have to solve word problems that involve "isosceles" they are going to know what it means.
>
> So, I spent much more time on the vocabulary this year. As a result, the students have started to interact more. There is a lot of communication that is going on. There is a lot of discussion going on.

Kaye continually worked to improve the strategic instructional task of mathematical communication by emphasizing the use of mathematical language.

Summary. The strategic instructional task of communication in Kaye's computational class, like that of the mathematical content, had been transformed. The students developed a common mathematical language through activities with manipulable materials and pictorial representations. This common language enabled them to think about and discuss the linkages that existed between the mathematical content and the concepts in each unit they studied. In addition, the methods of questioning, explaining, and discussing permitted the students and the teacher to engage in more math-focused interactions. Other techniques, such as error analysis and multiple representations of mathematical content, enabled students to develop flexibility in their thinking and discussing mathematical concepts and ideas. In her feedback to the students, Kaye stressed to them the importance of communication in learning and understanding mathematical concepts.

Kaye came to realize the importance of communication in the formation of mathematical concepts and conceptual thinking. Mathematical content without appropriate communication patterns would not likely lead to the students' development of mathematical concepts. She created linkages within the separate mathematical units and between them through the communication methods she implemented. She also worked to create linkages between mathematical communication and mathematical content that helped to promote the evolution of the conceptually oriented class.

The Evolution of the Social Organization

> What is happening now is that when the students are up at the front of the class putting their problems on the board, the other students are paying more attention.
>
> I think this is an important point and I hadn't looked at it from that view. I knew good things were happening, and that's what it is--the students are paying more attention.

> The students are trying to catch each other's mistakes and it is kind of fun. At this point it's not somebody trying to ridicule another classmate.
>
> (Pamela Kaye)

Kaye's statement described one of several techniques she used to transform the social organization of her class into one that reflected the goals and objectives of a conceptually oriented class. The social organization of the class consists of the methods that are used to organize students for instruction and tasks, the establishment of routines and procedures, and the techniques that support and maintain the goals and objectives of the class. The social organization of the computationally oriented class was characterized by a *notable lack* of whole-group instruction, lesson and unit planning, and student and teacher interest. It was a class where *substantial amounts* of time were given to mundane seatwork assignments, correcting endless amounts of student papers, and off-task student socializing. Social organization in the conceptually oriented class, in contrast, is characterized by *substantial amounts* of time-on-task behavior during whole-class instruction and a considerable amount of time given to lesson and unit planning. It is also a class where there is a *notable lack* of mundane individual seatwork assignments, endless quantities of papers to correct, and off-task socializing. In order for the computational class to evolve into the conceptually oriented class, modifications of this strategic instructional task would have to be employed. These modifications would have to encourage student involvement, enhance on-task behavior, and foster the development of mathematical ideas and concepts. In addition, modifications would have to be made in the strategic instructional task which linked the procedures of the class to the goals and objectives of the conceptually oriented class.

Descriptions and discussions of the methods and techniques Kaye used to link student activity to on-task mathematical behavior during direct

instruction and seatwork follow. Also descriptions of the methods she used to focus the students' attention on mathematical concepts and ideas to non-instructional activities are noted.

Methods to promote on-task behavior. Kaye implemented several methods to encourage student on-task behavior during whole-group instruction and seat-work. Three of these methods, student groups, student boardwork, and a review activity at the start of class, will be described and discussed.

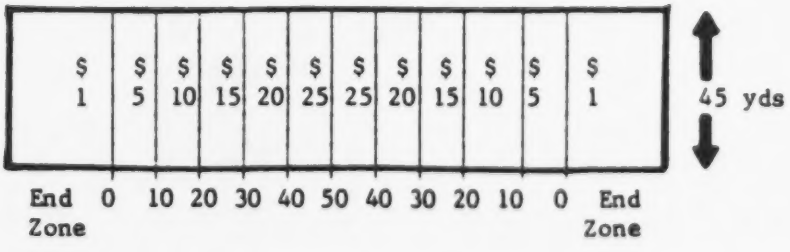
Kaye talked to the students on the first day of school about the kinds of group activities they would be working on throughout the year:

> Someone stopped by my room before school started and saw how I had arranged the seats. I told them that I was planning to have you work in groups this year. They said to me, "You gonna put General Math students in groups?! They'll copy each other!!" I told that person that I thought you probably wouldn't. By the end of the year you will like this class so much that you'll want to take it again.

Prior to the first day of school Kaye rearranged the desks in her room so the students were seated in clusters of four. An example of a group activity given to the students was one that required them to solve a problem related to a school project. The following observation describes the task given to the groups.

Kaye: You have a group project to do for the rest of the hour. The Athletic Boosters are trying to raise extra money and are selling square yards of the football field. I want you to figure out how much the boosters are going to make.

Kaye draws the following diagram on the board:



Kaye: The boosters are going to put the names of the buyers on a large drawing in the cafeteria. I want us to find out how many yards we can get out of the football field and how much money could be made. I want you to figure out how many square yards and the total amount of money this will generate.

Kaye writes the directions on the board:

1. Draw the figure
2. Figure yards total
3. Figure total amount of money

Kaye: I want you to put a group total of money on your paper. I want each of the groups to figure this out. Make sure you write your answer in a group.

After Kaye presented the problem and outlined the group activities, the students were given calculators and the rest of the period to solve the problem. At the end of the period, she recorded each group's answer on the chalkboard and selected one member from each group to explain to the class how his/her group arrived at the solution to the problem. Kaye continued to use groups for a variety of activities across the year. She reported on one outcome of groupwork in an interview.

Nason: How do the students relate to one another and to you in the general math class?

Kaye: I think they enter the classroom sometimes antagonistic of one another, not wanting anyone else to know how little they know about math. Mathematics is not what they are there for.

As the year progresses, I find them being more helpful to one another. Particularly this year, with them working in small groups, more of the students by the end of the year are cooperative to one another. I find them much more cooperative this year.

Kaye felt the students were on task more often and for longer periods of time when they were engaged in group activities than they were when they worked alone on individual assignments.

A method she used to capture the attention of the students and foster on-task behavior during whole-group direct instruction was to ask for student

volunteers. Student volunteers worked problems on the chalkboard with Kaye. The following observation selection describes a student working at the chalkboard.

Kaye: All right, I need a volunteer.

(Randy raises his hand and jumps up and down. Kaye calls on Donald to go to the chalkboard.)

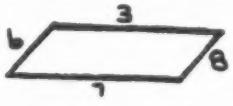
Kaye: Donald, I want you to draw me a parallelogram.

Donald: O.K. (He draws the following on the board.)

Kaye: Now, I want you to label it with numbers. (Donald fills in the numbers on the parallelogram.)

Kaye: Is there anything wrong?

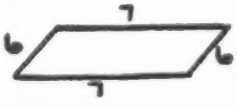
Mary: Yeah, the numbers should be different! (Donald looks at his drawing and changes the numbers.)



Donald: O.K. I knew the numbers should be different! I was just testing!!

Mary: Well, they're still not right! They need to be the same <u>length</u>!

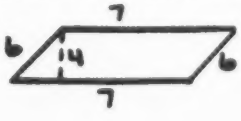
Donald: (Looks at the drawing) Oh. (He changes the numbers)



Kaye: All right, now I need you to write the area formula.

Donald: Oh, that's a hard one. Seven . . . no, I guess I forgot. It is something to do with adding these two together. . . .

Kaye: If you did this. . . . (She draws in a perpendicular line designating height.)



Donald: O.K. I remember now!
Donald writes in a 4 for the height and multiplies the base times the height.

Kaye: That's it!

Donald: No, it's not. It has something with adding in it.

Kaye: You're thinking about trapezoids.

The students in the class paid attention to Donald's boardwork problem, caught his mistake, and made him correct it. When Donald sat down more students were called on to work similar problems. Kaye continued calling on student volunteers to solve problems at the chalkboard because she thought it provided them with the opportunity to interact with each other as well as with the mathematical concepts and ideas. Boardwork also kept the attention of the students focused on the mathematical problems. The following example was taken from an observation in which Kaye used a student volunteer to solve a decimal problem.

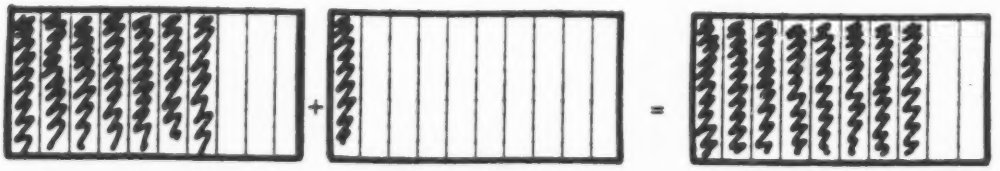
Kaye: All right Miss Freeman, show us a picture of the decimal seven-tenths added to the decimal one-tenth. (Karla goes to the board and picks up a piece of colored chalk to write her answer.)

Some students: All right! Colored chalk!!

Karla: (standing back and looking at her answer) Well, it's not quite a perfect box.

Randy: Do it in detail, do it in detail.

Karla draws the following:



Kaye: All right, what is this number then?

The students: Seven-tenths and one-tenth is eight-tenths.

Kaye: What would it look like if you wrote it vertically?

Randy: It would be just the same, eight-tenths which is really four-fifths.

The students watched and listened closely as Karla drew her figures. They participated in unison as they gave Kaye the answer to the problem. Kaye's view of using the students at the board had changed. She now realized it was a useful technique that promoted student on-task behavior and did not cause the students any embarrassment, as she had once thought.

Kaye promoted student on-task behavior by giving some review problems at the start of the class period. She implemented this technique because she wanted to focus student attention on mathematics at the very start of the class period. The following is a description of the first day this technique was implemented.

> The students are entering the room and chatting as the bell rings. Kaye walks in the room, closes the door behind her. The students are getting a piece of paper from her desk. One student is handing back some assignments.
>
> Kaye: Ladies and gentlemen, hush. Your first activity is a review of what you have just done. I am going to collect it as soon as I finish taking attendance, which should be at 8:05. (The students start working on the assignment. Kaye takes attendance.)
>
> Jessica: What's a perimeter?
>
> Kaye: What's the perimeter? I think I'm going to resign!
>
> When Kaye finishes with the attendance she collects the papers from the students. Most of the students have not had a chance to complete the review problems. About half the students are half the way through.

The reviews at the start of class frequently consisted of 4 or 5 problems from the previous day's lesson. A few days after Kaye implemented this activity the following observation was made.

> As the students enter the room they pick up a review sheet from Kaye's desk and take it to their seats and start working on it. The review contains four questions that review the work from yesterday's lesson.

This technique reduced the amount of time the students spent in off-task socializing, gave them a review of the previous day's content, and served to organize and prepare them for the daily lesson.

Kaye used groupwork, boardwork volunteers, and reviews at the start of the class as methods to increase the students attention and on-task behavior. These methods, once implemented, soon became routine and helped decrease the off-task socializing and contributed to the increase in participation and communication between students about the mathematical content.

Techniques used to facilitate the learning of mathematics. Student group activities, boardwork, and reviews were the modifications of the social organization instructional task Kaye used during seatwork, direct instruction, and at the start of the period. The purpose of using these methods was to more actively engage the students in thinking about and working on mathematics. There were other social organization techniques Kaye used to provide a mathematical focus to activities that took place outside of the direct instruction and seatwork times. These techniques included a daily and weekly agenda, more feedback about the students' mathematical progress, and more long-range planning of math lessons and units.

Kaye wrote a daily agenda on the chalkboard as an organizer for the students when they entered the classroom. The daily agenda included the date and the topic to be covered in the lesson for the day. It frequently included the materials they would use in the lesson. She also wrote a weekly agenda on the chalkboard, which gave students an opportunity to preview the weekly content, review the content covered during the week, and become aware of the conceptual linkages across the various mathematical topics.

A weekly grade report, the second social organization technique used by Kaye, provided a mathematical focus to the noninstructional activities of giving students feedback on their mathematical progress. She posted a weekly updated report sheet of student grades and absences in her three general mathematics classes. When the students entered the class they checked the computer printout posted on the bulletin board to find their class standing. The examples in Table 2 are part of the student standing reports for the first week in May and the first week in June.

Table 2
Students' Standing
in General Mathematics Class

Student's name	School grade	Points earned	Possible points	Percent average	Letter grade	Times absent
Standing on 5/6/85						
75 78 69 62 10	09	122	125	97.6	A+	0
86 78 65 57 10	09	126	130	96.9	A+	1
67 72 22 57 10	09	125	130	96.2	A	0
84 78 63 73 10	09	125	130	96.2	A	0
Standing on 6/6/85						
86 78 65 57 10	09	225	220	102.3	A+	1
67 72 22 57 10	09	221	220	100.5	A+	0
84 78 63 73 10	09	218	220	99.1	A+	0
75 78 69 62 10	09	212	215	98.6	A+	1

In the first week in May the average for Kaye's classes was 70.6% with the following: 13 *A's*, 20 *B's*, 20 *C's*, 14 *D's*, and 19 *F's*. By the first week in June, the average for her classes had risen to 74.2% with the following: 15 *A's*, 23 *B's*, 17 *C's*, 14 *D's* and 13 *F's*. Seven of the 19 students who had

earned *F's* during the first week in May moved up to the *C* and *D* range by the first week in June. Kaye believed that posting student standings encouraged her students to work harder and to get assignments completed and turned in. She said this provided them with the opportunity to follow their weekly mathematical progress and achievement gain throughout the grading periods. She also felt this helped her to manage student paperwork more efficiently. She said she was compelled to enter the scores from the students' work for the week into the computer before the following Monday morning because she had to post the standings on the bulletin board before class started. Since it only took a few minutes to enter the week's scores into her computer, the job was much easier than it had been in the past.

Planning for instruction was a technique to help Kaye become more organized in her thinking about the mathematical content within each unit and across the units. Prior to this, her yearly plans consisted of some general notions of the content she would like covered by the end of the first semester and the content that needed to be completed by the end of the year. She made little attempt to organize this content in a way that would help the students develop conceptual linkages and understandings across the units. Kaye said that now she thought a lot about the content she would teach during the following year, when she would teach the units, and how she would modify previously taught content before she taught it again. The following is her written description of the units she was planning to cover during the coming year.

General Math Schedule for the Year

First two weeks: Create decimeter cubes and inch cubes to discuss length, area in square units and volume. Can show one cubic meter with decimeters from all classes (first layer). Pretest with Shaw-Hiehle Test for Computation Skills.

Problem Solving: Units include guess and check, make a table, and look for patterns. Use Dolan and Williamson book, *Teaching Problem Solving Strategies*.

Next three to four weeks: Factors and Multiples Unit from the MSU Middle Grades Mathematics Project (MGMP).

Decimal materials as related to fraction strips and circles. (Allyn and Bacon Booklets)

Percent materials and applications.

Posttest with Shaw-Hiehle Test. Inform students of their improvement.

Second Semester: Continue with percent unit, followed with Probability Unit (MGMP) and then Similarity Unit (MGMP)

Consider following with more problem solving or the Mouse and Elephant Unit on scale factors from the MGMP materials. Follow with a unit on pre-algebra utilizing coordinate graphing material.

Kaye frequently talked to the students about the content they would be studying for the remainder of the semester or the year. She thought the technique of more thorough planning helped her think more about how to organize the students and instruction in ways that would provide a better conceptual focus to the mathematical content.

Summary. Kaye's general math class had been transformed through modifications of the strategic instructional task of social organization to reflect the goals and objectives of conceptually oriented instruction. Modifying the social organization encouraged student interest and involvement in mathematical tasks and content and provided a math focus to noninstructional class activities.

Kaye thought about the methods she employed that encouraged or sparked the evolution of the social organization. In the following interview, she talked about the outcomes of these methods after she had implemented them throughout the year.

Nason: Of the different teaching techniques you have tried, were there some that were more successful than others?

Kaye: I will continue to use group work. Using grouping to a greater extent than I have in the past has been helpful. I always allowed kids to work together, but it was much more structured this time.

There are some changes in the management of the class I have made that I will continue next year. A preview at the start of the class period will link things together and will cause the students to reflect back, "O.K. now, we've done this and this and today we're going to do this."

I think using a calendar on the blackboard of "Here's what we're going to do today and here's what we've done this week," with maybe a page number, or a worksheet. In terms of the students, it tells them, "Here's the direction that we're going to go." It tells the students, "Yes, we're going to do some math today."

I have made an attempt to get my paperwork better organized this year and I was able to accomplish that because I think it is really important for students to get feedback. That is something I need to continuously do in terms of my teaching the class.

Probably the biggest thing that needs to be done is to work on an overall plan and then try to figure out where things slide in from there. Whether, in fact, fractions, decimals, and percents really do belong all together.

Of the three strategic instructional tasks, improving the social organization was of least interest to Kaye. When she began to implement methods to improve the social organization she realized how useful they were in increasing students' on-task behavior in both the lessons and seatwork activities. In addition, these methods also provided a mathematical focus to the noninstructional class time. Kaye worked on modifying these methods to improve the social organization and looked for new ones to add when she began to think about her instruction for the coming year.

Summary of the Evolution

> My feelings about general mathematics have changed. You either do what you used to do three years ago and put out the worksheets and keep on doing that day after day--or you do what we've been doing.
>
> It is an either/or situation. Because once the students begin getting into discussions they are not satisfied anymore with doing worksheets.
>
> I wouldn't go back to doing worksheets anymore. That's really frustrating.

Pamela Kaye's class is now characterized as a class where learning of mathematical concepts and ideas is emphasized; a place where mathematical ideas and concepts are discussed; and an environment that fosters the learning and teaching of mathematics. Mathematical content/tasks evolved from an emphasis on the development of computational skills to a focus on the development of mathematical concepts and ideas. The quality and quantity of mathematical communication evolved from the sparse giving of directions and procedures into mathematical dialogues enriched with questions, discussions, and explanations. The social organization evolved from the organization of students and the establishment of routines and procedures that promoted the mass production of quantities of computed answers into an environment that in every aspect encouraged the development of mathematical thinking and understanding.

When asked at a teacher-researcher meeting to cite the rewards of teaching general mathematics using the modifications of the strategic instructional tasks, Kaye replied,

> When I think how hard my job is now, I sometimes think it would be a whole lot easier to say, "Here is how you add, folks." Then give them the worksheet with 50 problems on it. Then work one-on-one with each student.

It is a lot harder to be up front going at it for the entire class period. That's a lot more work for me. But it is a lot more enjoyable. It takes a lot more work to teach the class the way I'm teaching it now. But it is definitely more rewarding.

It is much more enjoyable this way.

A Conceptually Oriented Class

I think this class is just fine. In fact, if I could take it over for credit, I would!

(A Student)

A Typical Day

When the students entered Kaye's class, they picked up a piece of paper from her desk, took their seats, and began working on the review assignment which was written on the chalkboard.

8:00
The students have entered the room and are working on the review problems Kaye has written on the chalkboard:

Given 4 Blue, 2 Green, 2 White, 1 Yellow

Find:
1. P(B)=
2. P(G)=
3. P(W)=
4. P(Y)=
5. Sum of Probabilities=
6. How many W need to be added to get P(W)=1/2?

Some R, W, G

7. P(R)=1/3
   P(W)=1/4
   P(G)=___

2 Coins are flipped

8. Draw probability tree and list the outcomes
9. Find: P(HH)=
10. Find: P(at least 2 H)=
11. Find: P (at least 1 H)=

3 Chips labeled with letters A,B B,C C,D

12. Draw probability tree and list outcomes
13. Find: P(match)=
14. Find: P(at least 1 A)=

The class was working on a Probability Unit and this review covered material they had studied recently. Kaye used the problems in this review to help students understand the relationship of the data they gathered in the activities of this unit to the theoretical outcomes that were represented pictorially (with probability trees) and symbolically (expressed as rational numbers). Review activities at the start of the period involved the students in thinking about the mathematics they had just covered. These activities also gave the teacher the time to complete record keeping and attendance, to hand back corrected papers to the students, distribute materials needed for the daily lesson, and to work individually with students needing help.

8:07
As the students are working on their assignment Kaye hands back their papers from the assignment they did yesterday.

Tom and Jim are working on the review problems together and their results don't agree. They ask Kaye what the correct answer is.

Mary is showing Diane (who was absent yesterday) how to find the probability of getting a white marble.

Mary: Look here, what do you think the probability of getting a *W* would be here?

Kaye usually gave the students no more than 10 minutes for the daily review. However, since the review for today contained more problems than usual, she allowed the students an additional 10 minutes.

8:16
Kaye finishes taking attendance and circulates around the room checking on the students as they are working. She works with individual students who have questions.

Kaye hands back the papers from the assignment yesterday. When she finishes this she goes to the front of the room.

Kaye: All right, would you hand in your review sheets up to the front, please?

Tom: You mean we're not going to check them?

Kaye doesn't hear him and is erasing the review from the chalkboard. She then collects the papers and puts them on her desk.

Kaye started the daily lesson by reviewing material the students covered the previous day. During that lesson, they worked in pairs to collect data on two Dice Games and then were asked to determine which of the two games was fair and which was unfair. The students took turns rolling two dice. In the first game, after each roll the two numbers showing on the faces were *added*. In the second game, after each roll, the two numbers showing on the faces were *multiplied*. In either case, Player 1 was given one point if the sum (in game 1) or the product (in game 2) was even. Player 2 was awarded one point if the sum (in game 1) or the product (in game 2) was odd. Yesterday most of the students had only been able to play the two games and gather their data. Today, the instructional period was used for analyzing the results of the games.

8:20

Kaye: Yesterday when we finished class we looked at the total probabilities of something happening. Who can tell me what the class probabilities or the total sum was that we got when all the scores were added together? (Kaye has written on the chalkboard.)

Sums	Products
$P(E) = \frac{178}{316}$	$P(E) =$
$P(O) = \frac{138}{316}$	$P(O) =$

Kaye: Well that looks pretty uneven to me. Do you think this was a fair game?

The students: Uh, huh.

Kaye: Let's see what the decimal value of this (178/316) comes out to be (using a calculator to find the decimal value).

Other students in the class who have calculated the decimal value agree with Kaye's result.

Kaye: I'll tell you how you can figure out what the other value would be without using a calculator. (Kaye writes .80 on the board.)

Kaye: If I had .80, then you should get .20 for the other value. If I had .10, you should get .90 for the other value.

Some of the students are seeing the pattern and are able to give Kaye the value of the second number as soon as she finishes writing the first. (On the board Kaye has written the following.)

.20	.80
.10	.90
.25	
.74	

Kaye: If I had .25, you should get...

The students: .75.

Kaye: And if I have .74, then you should get...

The students: .26.

The students are by now aware that the sums of the two decimals should equal 1.00.

The students realized the sum of the two results of the addition game (178/316 or .56 and 138/316 or .44) equaled 1.00. Since the experimental results were nearly equal, the first game could be considered to be a fair game for both players. When the students compared their results from Game 1 (the sums) with those of Game 2 (the products), they easily determined that Game 2 gave an advantage to Player 2 and was an unfair game.

8:25

Kaye: Jane, what did you get for the P(E) of the products from your work yesterday?

Jane: 204/300 equals .68.

Kaye: So, the P(O) would be...96/300 and that would equal .32. It seems to me that the results are pretty close, right Ron?

Ron: Huh?

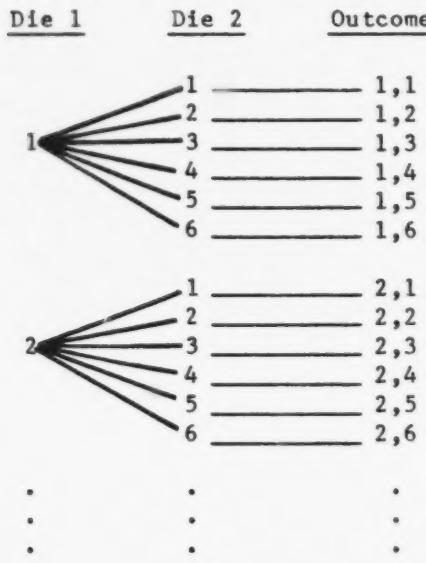
Kaye: Why do you think the game is unfair?

Ron: Because the answers aren't more even.

Kaye used a probability tree to show the students the theoretical outcomes of rolling a die. This helped them link the experimental results they obtained by playing the two games with the theoretical concepts of probability.

8:30

Kaye: All right, if we rolled a die, what kinds of outcomes could we get for the result? (Kaye writes on the board.)



Kaye shows the students how they could have obtained their answers on the work they did yesterday without rolling the dice (as they did in their probability experiment yesterday).

Kaye tells the students that if they continued recording their results this way it would take a lot of paper.

Kaye: You could do something to show these results in another way. You could do something you did in elementary school. (Kaye draws the following table on the board.)

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Jeff: Oh, a times chart!

Kaye: Yes, but we are going to add the numbers.

Mary: Do we have to write this on our papers?

Kaye: Turn over the papers I handed back to you today. You will see that there are charts printed on the backs. I want you to write the sums and the products on these charts.

The students start recording the sums and products in the charts printed on the back of the assignment sheet they did yesterday.

The charts are as follows:

Activity 3-2 Analyzing Two-Dice Games

Game 1
Sum

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Total Sums = _____
Total Number of Even Sums = _____
Total Number of Odd Sums = _____

P (Even Sum) = _____

P(Odd Sum) = _____

Game 2
Product

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Total Products = _____
Total Number of Even Products = _____
Total Number of Odd Products = _____

P(Even Product) = _____

P(Odd Product) = _____

Using familiar addition and multiplication charts, Kaye showed the students how the probability outcomes obtained by drawing probability trees could be more easily organized. The students completed the task of filling their charts within 5 minutes.

8:38
Most of the students have finished filling in their charts.

Kaye: All right, I need a volunteer to come up to the overhead and write their results for their *sums* chart on the transparency. (Ron volunteers and

starts to write his answers on the overhead. Ron finishes and Kaye asks for another volunteer.)

Kaye: All right, I need another volunteer to write the *products* on the overhead. (John volunteers and goes up to the overhead and writes his answers. Joe, Dick and Ron are collaborating on their answers as John continues to write his answers on the overhead.)

Kaye called on two students who had finished their work to write their answers on the overhead projector while the rest of the class completed their charts. When the two student volunteers were finished, Kaye reviewed the results with the class.

8:42
Kaye turns on the overhead light and turns down the lights in the room. The following results have been recorded on the transparency:

Activity 3-2 Analyzing Two-Dice Games

Game 1

Sums

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Game 2

Products

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Kaye: How many sums are there?

The students: Thirty-six.

Kaye: What is the total number of Evens?

The students: Eighteen.

Kaye: And there are 18 Odds. So the probability of getting an Even, Ron, would be what?

Ron: Eighteen over 36. And so would be the next one (referring to the probability of getting an Odd).

Kaye: Now, compare those results with the results you got when you did the experiment yesterday.

Jane: I told you I was right. It's half and half.

Kaye: That's right. Yesterday Jane told me that she thought the result would be half and half.

Kaye: Now look at the next chart and see what you got for the number of Evens and Odds.

The students: 27 over 36 for Evens, and 9 over 36 for the Odds.

Kaye: And what does that reduce to?

The students: Three-fourths and one-fourth.

Kaye: If you know that the first answer came out to be one-fourth, then what does the second outcome have to be?

The students: Three-fourths.

Kaye's continual questioning of the answers her students gave helped them make links between the probability outcomes and the part/whole relationship of fractions. She had the students work along with her on their papers as she continued asking questions.

8:45

Kaye: What is the probability of getting a sum of three? I want you to write on your papers a *P* and then in brackets, *a sum of three*. (Kaye writes on the board what she wants the students to put on their papers.)

P (a sum of 3)

Joe: On which paper?

Kaye: On the Sum side of the charts.

Joe: The answer would be 2.

Kaye: Two out of how many outcomes?

Joe: It would be 36.

Kaye: So?

Joe: It would be 2 out of 36.

Kaye: What is the probability of getting a 9?

John: Four out of 36.

Kaye: Right, can that be reduced?

John: Yes, one-ninth.

Kaye: What is the probability of getting a sum greater than 7? Jim?

Jim: One-third? I don't know.

Kaye: Well tell us how you did that.

Jim: I guess that I counted, like 8 . . 9 . . 10 . . .

Kaye: Well, how many are there then?

Jim: Fifteen.

Kaye: Fifteen out of how many altogether?

Jim: Thirty-six.

Kaye: Then how would you write the probability?

Jim: Fifteen out of 36, 15 over 36.

Kaye: And that reduces to?

The students: Five-twelfths.

Joe, John, or Jim were not permitted to give simple one word responses. Instead, Kaye persisted with a series of questions for each student until certain they, and other students, understood the concepts.

With 2 minutes left in the period, the students started to discuss the outcomes of the second chart on finding the products. As with the discussion of the sums chart, Kaye's questions on the products chart focused on probability outcomes as well as on rational numbers.

8:53
Kaye: How many products of 12 do you have in your charts? (Kaye and the students are working on the product chart.)

The students: Four.

Kaye: Out of how many?

The students: Thirty-six.

Kaye: And that reduces to?

The students: One-ninth.

Kaye: How about the numbers that give a product of 16?

The students: Only one.

Kaye: Right. Now how about the products that are greater than 12?

The students: 13.

The Transformed Class

This general math class of Kaye's clearly has a different orientation than the one portrayed two years earlier--before project intervention activities began. Kaye, in collaboration with researchers, had transformed her general math class from one that was computation-oriented to one that was concept-oriented. She accomplished the transformation by modifying her thoughts and actions--in a reflective and systematic manner--relative to decisions and judgments about the three strategic instructional tasks:

- Selection of mathematical content/tasks,
- Communication of the content,
- Organization for implementing and communicating the content.

Modification of mathematical content and tasks. The linkages that were made between the probability outcomes, fractions, and decimals illustrated Kaye's attempt to help students become aware of the mathematical interrelationships across several different math units. The introduction of the unit on probability provided her students with an opportunity to experience new mathematical concepts that were interesting and challenging. Kaye's use of activities involving manipulable and pictorial representations enhanced the development of the students' concept of probability as well as enhancing their conceptual understandings of fractions and decimals.

Increase in quality and quantity of mathematical communication. Kaye's continual questioning of her students' thinking and their answers, her persistence in having the students give complete explanations, and her emphasis on the use of mathematical vocabulary (i.e. products, sums, and probability outcomes) increased both the quality and quantity of communication about the content. The improvement in the patterns of communication contributed to the students' ability to think and talk about mathematical concepts and relationships.

Enhancement of mathematical learning and instruction through social organization. Kaye used the following methods to improve the social organization of the class: the start of class review, the organization of materials, increased lesson planning and preparation, student volunteers, the use of the overhead, and students working in pairs and groups. These methods enhanced the mathematical learning and instruction indirectly because they directed student attention to the daily mathematical content and encouraged on-task behavior throughout the class period.

The Consequences

> Overall on a scale of 1 to 10, this class gets a 9 becaus [*sic*] no class is perfact [*sic*].
>
> (A student)

This section will consider consequences of conceptual orientation for Kaye, her students, and in-class activity. Kaye's knowledge and beliefs about general math, the students, and her instruction had been transformed. Mathematical achievement, confidence, and mathematical attitudes of her students had been affected by her evolution of thought and action.

What were the consequences for the teacher? Kaye clung to the computational competency goal. However, she now thought computational competency could be attained by focusing instruction of conceptual understanding of the mathematical content. In an interview at the end of the project Kaye's changed views about the content of general mathematics are apparent:

Nason: When you think of general mathematics, what mathematics do you think of?

Kaye: Well, way back before the project, I talked about add, subtract, multiply, divide, whole numbers, decimals, and fractions. I still think that, but I guess in a different context. The mathematics I am doing now is more conceptual.

I am still looking at basic skills, but I am trying to build to some higher level [conceptual]. What I am trying to build to is how mathematics is applied to other areas.

Through the use of things like similarity, probability and so forth, the mathematics has become more conceptual. I still think in the end the goal I am going after is computational skills, but they are achieved through conceptual understandings.

Kaye identified new, interesting, and challenging content, such as the units on similarity and probability that could be used to develop the conceptual understandings that would lead to computational competency.

At the start of the project, she believed that if she could get students to have confidence in themselves first they would then have the confidence to be successful in mathematics. She now knew it was the reverse--it was their mathematical success that enhanced their self-confidence. Kaye described her general mathematics students differently in the final interview than she did in the first interview.

Nason: Describe as a group, the students who usually make up the general math class.

Kaye: I think they are students who have just missed out in math somewhere and need help getting over the hump. I think many of them believe they can't learn, especially when they first come in here.

I think that they are average students and they are fun-loving students.

Nason: Is that different from the way you would have described them three years ago?

Kaye: Probably. I think the thing that was stuck in my mind back then was the problem students that were in the general math class. Those were the students with various learning, emotional, and discipline problems. Those were the students I focused on. At least the way general math had been taught in the past.

Now there is a chance for the kids who are enjoyable to come to the front. When they see they can succeed, they get more ambitious and a lot more fun to work with.

Kaye still believed drill and practice was important in the development of computational skills. However, there was a substantial change reflected in an interview in the way she thought drill and practice should be carried out.

Nason: How important do you think drill and practice is in general mathematics?

Kaye: I think it is very important, but not in the same way I used to. I think it is important to spread a few problems a day over a long period of time, not doing 60 of one kind at one sitting.

Drill and practice is much more more valuable if couched in another setting. Once students have an idea of what fractions are all about, they can get the drill and practice they need in some other content, like probability and similarity.

She now realized that a few review problems given daily over an extended period of time was a more successful means of providing the students with computational reviews than were the lengthy seatwork assignments she had assigned earlier. She also knew that drill and practice could be camouflaged in other mathematical content areas.

Kaye once thought the way to improve general mathematics would be to simply use more manipulable materials (in spite of the students' resistance), to eliminate grades, and instruct each student individually. When asked her

thoughts now about improving general mathematics she centered on methods used to enhance the conceptual development of mathematics.

Nason: How could general mathematics be improved?

Kaye: One of the things is more small-group activities that require group participation to complete. I think that a continual effort needs to be made on my part to find manipulatives that demonstrate concretely the various concepts.

I definitely think that conceptual instruction is definitely the direction we need to go. The other thing is the continuation of questioning...that's the key to it right there.

Nason: How could these changes be accomplished?

Kaye: Well, in terms of the concrete kinds of things, I think there needs to be more talking with other teachers. The questioning technique is something each teacher needs to work on their own. If it could be done, having someone come in your classroom and observe what you are doing.

Nason: To give you feedback?

Kaye: Yes.

The result of these changes would be to improve the general mathematics classroom and improve the students' ability to deal with math. I am talking about their math skills. I personally would like to see all of their skill levels, you are going to enhance all the other things.

Kaye now believed that using the social organization, enhancing the mathematical communication, emphasizing conceptually oriented instruction, and collaborating with other teachers would improve general math. These responses differed greatly from those she gave at the outset.

In reflecting on efforts to transform her instructional orientation, she assumed success in increasing the quality and quantity of *communication* about the content. She also thought she had been successful in implementing some *social organization* techniques, such as planning, managing paperwork, and keeping a daily agenda on the chalkboard; however, she felt less successful in

using small-group activities and making use of daily logs to record instructional outcomes. She felt successful in modifying the *mathematical content* by adding new instructional units and changing all her previously taught units to now emphasize conceptual understanding.

At a teacher-researcher meeting she revealed some of her changed beliefs when questioned by project coordinator Jim Buschman.

Buschman: Realistically, how much reward is there in doing what you are doing? And where is it?

Kaye: That's a toughie!

Buschman: Yes.

Kaye: One of the rewards for me is when I get to the end of the year and the kids I have said three or four times in the last semester, "You need to take algebra next year. You know you are really going to need it for what you want to do." It's a big payoff for me when they come back in September and tell me they are taking algebra and they tell me, "Say, that's real easy!" That didn't happen very often when I was teaching in a more traditional fashion. There's a lot more of it happening now.

I remember at the beginning of the project saying lots about how getting those students comfortable with one another and having some confidence in themselves, to me, was as important as teaching them some math skills.

I was not sure that I was ever convinced that you could do both. Now, I think I'm getting to know those students through the mathematics I am teaching. Some kind of relationship is happening between us. There is more communication about the math content, and there is more communication period.

Kaye believed that she had become a better teacher, that the classroom had been transformed into a place for the learning and teaching of mathematics and, most importantly, for her, that the students were finally experiencing mathematical success and achievement.

What were the in-class consequences?

In Kaye's computational class there was little communication about mathematics, a lot of seatwork, and frequent off-task socializing. Classroom observations captured the evolution of the computational class as it became a conceptually oriented class in two ways: First, by *measuring the amount of time* the students and Kaye spent in various activities throughout each class period; and second, by *describing the nature* of these activities and how they had changed. The periods of both the computational and the conceptual classes consisted of some form of whole-class direct instruction, a seatwork assignment, and other activities not related to the daily mathematics instruction (nonmathematical activities). These activities are defined below:

Definition of Activities

Direct instruction

Lesson Development: Whole-class direct instruction of the daily lesson.

Review: Whole-class discussion and dialogues of previously taught content.

Checking: Whole-class discussions of the results of the assignments.

Seatwork

Practice: Individual or small-group work which was related to the content of the daily lesson.

Review: Individual or small-group work which reviewed the content of previous lessons.

Test: Individual test or quiz.

Nonmathematical

Management: Record keeping, distributing or collecting materials or student work.

Socializing: Interactions between the students or the teacher and the students which are non-math-related.

Analysis of the classroom observations of Kaye's *computationally oriented class*, *the intervention period*, and *the conceptually oriented class*, provided information on the average amount of time spent in each class period on direct instruction, seatwork, and other nonmathematical activities. Table 3 represents three different periods: the first, Kaye's *computational class* (classroom observations made prior to the start of the transformation activities); the second, the *intervention* (observations of the first year Kaye implemented the instructional improvement strategies); and third, the *conceptual class* (observations of the second year of Kaye's implementation and refinement of the instructional improvement strategies. There was a *168% increase* in the amount of time spent in direct instruction activities; a *29% decrease* in the amount of time spent in seatwork activities; and a *49% decrease* in the amount of time spent on nonmathematical activities from the computational class to the conceptual class periods. In general, more time was spent in direct instruction on the mathematical content and less time was spent in seatwork and other nonmathematical activities in the conceptually oriented class than in the computationally oriented class. The nature of class activities in the computational class and the conceptual class are contrasted below.

Direct Instruction:

The Computational Class

There were no reviews of previous content. Lesson development consisted of Kaye's directions on how to work the problems on the daily assignment. Checking the daily work included Kaye's rapid reading of the answers to the assignment.

The Conceptual Class

An oral review including discussions, questions, and explanations of the previous day's work preceded the daily lesson. Lesson development included dialogues, questions and explanations. Student volunteers were used at the chalkboard or overhead, and during controlled practice activities. Checking included error analysis and multiple ways the students were thinking about the problems' solutions.

Table 3

The Flow of Classroom Activity Across the Instructional Evolution in Pamela Kaye's General Mathematics Classes

Activity	Percent of Time Spent Per Class Period		
	Year 1 Computational Class	Years 2 and 3 Intervention	Conceptual Class
	n=8	n=46	n=62
Direct instruction	16.7%	39.0%	44.8%
Review	(0.0%)	(4.6%)	(5.5%)
Lesson development	(14.2%)	(21.4%)	(25.9%)
Checking	(2.5%)	(13.0%)	(13.4%)
Seatwork	63.3%	48.4%	45.0%
Review	(0.0%)	(8.1%)	(12.2%)
Practice	(60.2%)	(32.8%)	(24.2%)
Test	(3.1%)	(7.5%)	(8.6%)
Nonmathematical	20.0%	12.7%	10.2%
Managing	(9.2%)	(10.8%)	(9.06%)
Socializing	(10.8%)	(1.9%)	(1.1%)

Seatwork:

The Computational Class

There were no seatwork problems which reviewed previously learned content. Assignments were numerous computational problems. The students worked on whole numbers, decimals, and fractions. Tests were computational assessments of the content which was just covered.

The Conceptual Class

There were some review problems of content recently covered at the start of each period. Seatwork activities integrated manipulable materials, pictorial representations, and symbolic abstractions to enhance student understanding and conceptual development. New units were added and group-work activities were used. Tests assessed students' understanding of content just covered and their cumulative learnings. The tests measured both conceptual development and computational competence.

Nonmathematical activities:

The Computational Class

Management consisted of maintaining class records, collecting and distributing materials for the lesson, and dealing with the individual problems of students. Time was given for the students to socialize with each other at the start and end of each period.

The Conceptual Class

Management of records, distributing materials for the daily lesson, collecting student work, and dealing with individual student problems were still carried out during this period. The students, however, remained on-task at the beginning and at the end of the periods. Kaye expected them to work for the entire hour. Only occasionally was time given to the students to chat at the end of the class.

The computational/conceptual contrasts depict significant changes in the quality of class activities. Only *management* remained relatively unchanged across the periods. In the evidence indicated there were substantial increases in the amount of time spent on-task and in direct instruction as well as noted improvements in the nature of the activities.

What were the consequences for students? Kaye used a computational test as an indicator of the mathematical progress her students made across the year. The Shaw-Hiehle Computation Test was administered each year (1982-84) in September, January, and June. A mean percent of the correct items on the five subtests and the total test was calculated for both the pretests and posttests. This was also done for two general math classes of another teacher in Kaye's school. Kaye had 121 students that took this test (over 2 years), the other teacher had 50 students (over 1 year). The results of these tests are included in Table 4 below:

Table 4

Shaw-Hiehle Computation Test

(Mean Percents)

Arborville High School's General Math classes	SUBTESTS					TOTAL TEST	
	Whole Nos. 20 items	Fractions 10 items	Decimals 10 items	Percents 10 items	Practical Problems 10 items	60 items	G.E.[a]
Pamela Kaye							
Pretest	77.3%	28.0%	52.5%	15.4%	38.8%	47.4%	6.6
Posttest	88.3%	61.7%	77.3%	46.5%	57.7%	68.0%	8.6
Another Teacher							
Pretest	77.8%	23.4%	55.4%	20.1%	40.9%	50.2%	6.9
Posttest	81.5%	49.3%	67.6%	30.5%	43.2%	58.6%	7.7

[a]Grade level equivalent

The students in Kaye's classes showed more gains than did the students in the other classes. Most striking were the gains made in the fraction, percent, and practical problems subtests.

A second analysis of the test items on the Shaw-Hiehle Computation Test provided a measure of student effort. Improvement in mathematical confidence was determined if a student took the time to work the problems on the test. Students who attempted more problems (items) on the posttest than on the pretest were assumed to have gained more confidence in their ability to answer those items correctly. Since answers on the Shaw-Hiehle Computation Test had to be calculated by hand, a student either answered the item correctly, incorrectly, or did not attempt to answer the item. In obtaining a student effort score, the number of items attempted (whether correct or incorrect) were counted. The percent of items attempted on the pretest and posttest are included in Table 5.

Table 5

Shaw-Hiehle Computation Test

(Percent of Items Attempted)

Arborville High School's General Math Classes	SUBTESTS					TOTAL TEST	
	Whole Nos. 20 items	Fractions 10 items	Decimals 10 items	Percents 10 items	Practical Problems 10 items	60 items	% gain
Pamela Kaye							
Pretest	96.7%	86.3%	94.6%	51.6%	75.2%	83.7%	
Posttest	99.9%	98.2%	99.2%	94.5%	96.6%	98.1%	+ 14.7%
Another Teacher							
Pretest	96.8%	89.2%	93.5%	50.0%	71.1%	82.9%	
Posttest	95.8%	84.9%	92.2%	52.2%	71.6%	82.1%	- 0.1%

The results indicated students in Kaye's general mathematics classes attempted to work more items on the posttest than did students in the other teacher's classes. The percent of items tried on the posttest in the fractions, percents, and practical problems subtests is striking between the two sets of classes. It should be noted that the students in the other general math classes spent the year working on computational reviews of basic arithmetic problems.

Kaye wanted to find out what the students thought they learned and what they thought about the class. She asked them to respond to the following:

1. Tell me what (if anything) *new* things you learned this year.
2. Tell me if there was anything you had before but didn't *understand* -- and now you do.
3. Please tell me any changes you think should be made for next year, or any other suggestions you might have.

The students were free to respond as they wished since they were asked not to write their names on their responses. Some of their responses (typed as written), were:

> I leard about LCM & GCF and I leard a little more about fraction and reduseing them. And you explane things others teachers give you a book and page number and tell you to read the directions. I think your a good teacher and I have no suggestions because you are doing a good job.

> I new most all of the things we did But neve understood it real well. You made things clear and helped me all of the time i learned more this semester than i did all last year
>
> yours turley
> guess

> I learned what GCF & LCM were. How to read decimals, How to change decimals into fractions and %. How to +, -, x, +, fractions. I neve understood any of this, that was why I hated math. our teacher would give us page numbers and say good luck. It is more if someone explains it to you. The only suggestions I have are to make more teachers do it this way. But don't have them start doing it in 9th grade. Start in lower grades like 5th or 6th so ya know what your doing.

Almost everything. The math teacher last year spent 2 days on 1 thing that you would have spent 2 weeks on.

I never really understood math. But all it really took was a good teacher. I know a lot more than befor I came in here.

In analyzing the total set of responses students' comments were categorized. The results of this end-of-the-year survey are included in Table 6.

Table 6

Year-End Student Questionnaire

Question 1: Tell me what (if anything) new you learned this year.		Question 2: Tell me if there was anything you had before but didn't understand --and now you do.		Question 3: Tell me any changes you think should be made for next year or any other suggestions you might have.	
Responses		Responses		Responses	
A lot	16	A lot	6	No change	16
Percent	17	Fractions	18	Good class, good teacher	12
Fractions	15	Percents	14	Less work	5
Integers	10	Integers	6	More groups	3
Probability	9	Decimals	5	More fractions, similarity, and probability	3
Decimals	9	Equations	4	Less boardwork	2
Similarity	4	Probability	4	More dittoes	2
Graphs	4	Perimeter	1	Computation only	1
Problem solving	2	Graphs	1	No percents	2
Area and perimeter	1	Problem solving	1	Less lecturing	1
Angles	1	Geometric Shapes	1		
Nothing	2	Nothing	5		

The analysis indicated that many students said there was *a lot* that was new for them this year. Of these *new* learnings, 32 students said they learned about fractions and percents--although they had been in math classes for at least 8 years. There were also 32 students who mentioned they *now understood* fractions and percents. When asked to describe any changes they would make in the class, 28 said that there should be *no change* or that it was a good class

and Kaye was a good teacher. From these results it seemed that the students in Kaye's classes liked being there and felt it was a good place to be. This is quite a contrast when one considers Kaye's view of general math students' attitude at the beginning of the project:

> If they had their druthers, they wouldn't be there. It has nothing to do with me, it's just that it's general math class.

and the attitude of a student at the end of the year depicted by the following:

> I think this course is just fine. In fact, if I could take it over for credit I would.

Conclusion

The consequences of the transformation from a computationally oriented class to a conceptually oriented one for the teacher, the class, and the students have been noted above. For Kaye, teaching general mathematics had become a rewarding experience. For her students, the class was a challenging, interesting, and rewarding place to be. Modifying the strategic instructional tasks of the computational class in an evolutionary way improved the quality of instruction which in turn resulted in more desirable learning via the conceptual class.

Conclusions

The question that drove our General Mathematics Project was "Can interventions be designed for ninth-grade general mathematics students that concomitantly alleviate constraints and ameloriate learning opportunity and teaching conditions?" We identified three strategic instructional tasks which, if modified, might enable us to respond to the question affirmatively: selecting mathematical content/tasks, communicating that content to students, and organizing students for the tasks. Once identified we set out to

modify/intervene with teachers' usual mode of implementing these respective instructional tasks. In the spirit of the IRT we sought a change in both the teachers' thoughts and the teachers' actions in order to improve the quality of their instruction.

Although the three case studies focused on the teachers' implementation of the modified instructional tasks, the General Mathematics Project had changed the teachers' thoughts and actions relative to those strategic instructional tasks. The genesis of the thought change came from readings which were followed by discussions with researchers and other project teachers. Similarly, the genesis for the change in teacher action came from the project's support and assistance to the teachers as they systematically planned the interventions and the classroom consultation provided as they tried these interventions in their general math classes.

What We Learned

1. Teachers' altered thoughts about and actions about the three strategic instructional tasks can improve general mathematics classes.

In the third year of the project we found that the collaborating teachers were enacting the three strategic instructional tasks in ways that fostered general mathematics students' thinking about and understanding of mathematical concepts, principles, and generalizations. Based on this type of student participation, teachers' assessments, and our own observations we judged that in each target class the students and the teacher experienced educational success--a clear improvement! Further, this improvement could be traced to readings[8] and deliberations related to the strategic instructional tasks. These were:

---

[8]The readings related to modifying the strategic instructional tasks are referenced in Appendices A, B, and C.

Strategic Instructional Task	Topics of Selected Readings
Communication about the mathematical content	Questioning, clarity, listening, responding, feedback, wait time
Using the social organization to improve math instruction	Classroom organization and management, student groups, instructional organization
Modifying the math content/tasks	Teaching for conceptual understanding, concrete-pictorial-abstract linkages, new topics of content to enhance mathematical learning and instruction

2. Habitual consideration of all three strategies and their unification for every lesson is necessary to optimize instruction.

At the outset we surmised that modifying the set of three tasks was necessary and sufficient to improve general mathematics although we assumed teachers would probably focus on changing only one--at most two--task(s) at a time. Goodlad (1984) noted that significant improvements in student learning would occur when instructional approaches were implemented in several areas:

> No single variable in itself appears sufficiently powerful to influence student learning significantly. Rather, it appears that each of a number of approaches carries some weight, and orchestrated together, they can add up to a significant difference. One of these approaches involves arranging and rearranging instructional groups and methods to achieve changing purposes--for example, shifting from large group instruction involving lecturing to small groups necessitating student interaction.
>
> A second has to do with variability--varying the focus of learning from textbooks, to films, to field trips, to library research in order to assure different avenues to the same learnings.
>
> A third approach, growing in recognized importance, stresses clarity of instructions and support for and feedback to the learner: expectations are clear; good performance is praised; errors or faulty approaches are pointed out just as quickly as possible; or a learner having unusual trouble with particular procedures being used is provided with an alternative method to the one used with the total group (p. 104).

Project results tend to bear out both Goodlad's and our early conjectures, and we suspect that some of the unevenness in improvement across

teachers can be attributed to the degree each modified the three respective tasks. For example, the teacher whose class's achievement record was uneven across semesters did not habitually consider the use of social organization. Another who hesitated to regularly risk using her own judgment in task selection frequently spent excessive amounts of time talking about an insignificant or minor attribute of the concept under consideration. Toward the end of the project the teachers became aware of the interdependencies of the tasks. Even when their instruction might not reflect this awareness, their talk about the success or problems of a particular lesson would do so. It was common to hear them say, "I don't think that task was appropriate for group work" or "I should have had the students work in pairs so there would have been more talk about the proportional relationship of the sides of similar triangles."

3. Teachers' modifications of the instructional tasks were idiosyncratic and partial (i.e., they never prescriptively accepted a recommended change as presented--it was always adapted to their present habits in some way--it had to fit in).

Just as the teachers resisted implementing all three of the improvement strategies simultaneously, they also resisted implementing methods or strategies from the readings or research results that they saw as simple prescriptions or directions for effective teacher behavior or instruction. Instead, after reading and discussing these selections, they chose certain portions they deemed important and modified them to fit their instructional patterns. For example, one teacher found Good and Grouws's (1979) *controlled practice during direct instruction* to be a useful and effective instructional method and continued to use it in her class. In contrast, she did not find the homework component of the Good and Grouws Instructional Model either a useful or effective instructional tool that would enhance student learning--so she simply chose not to implement this.

4. In order for any modification of the strategic instructional tasks to become a part of a teacher's instructional mode, the teacher had to (a) try the modification in the class, (b) become aware of the effect of the modification on the students and their learning, (c) reflect on the trial of the modification with the project staff, (d) improve the modification, and (e) try it in his/her class again.

As modifications of the strategic tasks were implemented we studied the ways in which ones that had been successfully implemented were different from those that had been tried and then dropped. We learned the modifications judged by the teachers as unsuccessful were those that they thought too difficult to be implemented or did not promote positive student outcomes. These modifications were dropped by the teachers after several attempts had been made to implement them. In contrast, the modifications of the strategic instructional tasks that became a part of the teachers' instructional repertoire had been tried, reflected upon, discussed, revised, and retried. We found those modifications that were successfully implemented fit closer with the teacher's instruction, produced more favorable student results, or were more easily modified to fit the needs of students, the teacher, and the class. For example, the strategy of using controlled practice during whole-group instruction proved to be successful for one of our teachers because the first time she tried it she noticed an immediate increase in communication about the content that occurred among her students. She revised the strategy to include student volunteers and some group work and then tried it again. She believed this revised version of the strategy contributed to even greater student achievement and participation. One strategy that was not successfully implemented by this same teacher was the grouping of students for seatwork. Although she tried to group students several times across the duration of the project, she was uncomfortable with assigning students to groups and claimed she could not find enough group tasks to warrant using permanent groupings for

seatwork. After several trials she stopped using grouping as a strategy to improve the social organization of the class.

5. Teachers' conceptually oriented instruction via modification of the three strategic instructional tasks advanced students' computational competence.

The student achievement data and the related information acquired from the teachers--project and nonproject--regarding what they taught and their predictions of student performance on particular test items was analyzed in the context of the concept-oriented versus computation-oriented instruction.

To assess the effects of the instructional changes that teachers made on the achievement of their students, the class means on the Stanford Diagnostic Mathematics Test (pretest, interim, and posttest) were compared by total test and by subtest using ANOVA procedures. The results indicated that no significant differences between schools were found, but significant differences existed within each school between the students' achievement in the project teachers' classes and those in the nonproject classes. The major reason for this difference was that the students in the project teachers' classes scored much higher on the computation subtest than did the students in the nonproject classes.

Although at first glance this finding may seem difficult to explain, our descriptive and field data offer a plausible explanation. During the 1983-1984 school year we interviewed the three project and three nonproject teachers to ascertain what content they covered in their classes. Additionally, the worksheets or textbook problems assigned to students were collected to analyze the kinds of mathematical activities the students were typically engaged in (e.g., computation, problem solving, estimating, etc.). During these last six months the information collected from each teacher was analyzed using an adaptation of a taxonomy developed for this purpose (Freeman et al.,

1980). The two-dimensional taxonomy captured the intent of instruction (i.e., to teach concepts, computation, problem solving), as well as the mathematical content areas presented to the students (e.g., whole numbers, decimals, algebra, etc.).

The results revealed that the project teachers spent *less* time on computation than did their counterparts. In fact, the nonproject teachers spent at least 10% more of their total available instructional time (or approximately 18 more class hours) on computation. Thus, the project teachers' students could not have fared better on computation because they devoted more instructional time to it. The classroom observational data clearly show that project teachers spent more of their efforts toward building a conceptual mathematical base for their students. These teachers came to value conceptual knowledge. Thus, they strove to increase their students' understanding of mathematics as well as increase their computational skills.

These findings are most heartening because they clearly demonstrate that teachers can and do positively change their academic goals and instructional practices for students. Doyle (1983) cut to the heart of the problem when he stated:

> Some tasks, especially those which involve understanding and higher level cognitive processes, are difficult for teachers and students to accomplish in classrooms. In attempting to accomplish such tasks, students face ambiguity and risk generated by the accountability system. Teachers, in turn, face complex management problems resulting from delays and slowdowns and from the fact that a significant portion of the students may not be able to accomplish the assigned work. As tasks move toward memory or routine algorithms, these problems are reduced substantially. The central point is that the type of tasks which cognitive psychology suggests will have the greatest long-term consequences for improving the quality of academic work are precisely those which are the most difficult to install in classrooms. (p. 186)

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Appendix A

Improving the Quality and Quantity of Communication About Mathematics Content: Readings

Kaye believed the literature that focused on improving the quality and quantity of mathematical communication had significantly contributed to the development of the instructional methods she used to foster the communication evolution. The readings below are those which she thought were the most significant. Brief descriptions of the reasons why she thought they were helpful are also included. Other selected readings studied by the project staff related to improving communication are listed.

1. Rudnitsky's (1981) "Talking Mathematics with Children" discussed the value of dialogues with children in helping the teacher understand what the child knew. It emphasized the diagnostic value in talking mathematics with children.

2. Driscoll's (1983) "Communicating Mathematics" considered the significance of the language of mathematics and effective communication. He concluded that there were teacher behaviors (i.e., monitoring and listening) that would promote such effective mathematical communication.

3. Jencks's (1980) "Why Blame the Kids? We Teach Mistakes!" discussed the misconceptions of children's thoughts about fundamental arithmetic operations. He emphasized teachers should focus on teaching for conceptual understandings of the arithmetic operations in order to help children guide their thinking.

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Appendix B

Using the Social Organization of the Classrooms to Facilitate the Learning of Mathematics: Readings

The methods and techniques which were implemented to transform the social organization of Kaye's computational class were developed from the project-related activities. There were several readings from the selected literature on improving the social organization of the class Kaye found useful in helping her think about the things she could do to improve this area. These readings and brief summaries of the ideas Kaye found useful are as follows. Other selected readings studied by the project staff are included.

1. Fisher and Berliner's (1981) "Teaching Behavior, Academic Learning Time and Student Achievement" suggested that small group work provided a useful compromise for individualizing content, maintaining efficiency and task engagement, and providing social experiences.

2. Slavin's (1978) *Using Student Team Learning* noted that heterogeneous student groups promoted greater on-task behavior, higher academic achievement and cooperation than did situations where these groups were not used.

3. Good and Grouws's (1979) "The Missouri Mathematics Effectiveness Project: An Experimental Study in Fourth-Grade Classrooms" reported it was possible to improve student performance in mathematics through an organized system of instruction. A summary of "Instructional Behaviors" used by teachers in their study included: daily review, lesson development, seatwork, and homework.

4. Emmer and Evertson's (1981) *Effective Classroom Management at the Beginning of the School Year in Junior High Classes* noted that more effective managers had a more workable system of rules, monitored student behavior more closely, were more task-oriented, gave clearer directions, and actively instructed the whole class more often than having students do seatwork.

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Appendix C

Modifying the Content/Tasks of General Mathematics to Improve Learning and Teaching: Readings

The content evolution progressed through project-related activities which made Kaye aware of the changes she could make to improve the content and tasks in general mathematics. Among these activities were reviewing the literature related to improving the content/tasks of general mathematics, collaboration with the project's teachers and researchers, and implementing new or modified content and tasks in her general math classes.

Kaye found several readings from the project's selected literature helpful to her as she modified the mathematical content. The ideas she extracted from these readings were incorporated into her instructional modifications. The readings which were most significant for Kaye and her reasons for their importance are summarized below. Other selected readings are also referenced in this appendix.

1. Driscoll's (1983) "Understanding Fractions: A Prerequisite for Success in Secondary School Mathematics" noted that students did not see the flexible nature of fractions, expressed as measures, quotients, ratios, or operators. Teachers must encourage students to verbalize and engage in classroom dialogues to develop a full understanding of fractions.

2. Berman and Friederwitzer's (1983) "Teaching Fractions Without Numbers" emphasized the importance of using concrete materials during the development of the concept of fractions. The use of fractional circles was suggested as a way to broaden the concept development of fractions.

3. Carpenter's (1980) "N.A.E.P. Note: Problem Solving" recommended that specific attention should be given to the teaching of problem solving strategies. In addition, problem solving should be an integral part of all instruction, new mathematical topics should be cast in a problem-solving framework, and students should be guided into problem solving by the teacher asking a number of unobtrusive questions.

4. Driscoll's (1983) "Estimation: A Prerequisite for Success in Secondary School Mathematics" suggested teachers teach students to value estimates in their own right as distinct from exact

answers. Estimation skills should be taught on a regular basis.

5. Anderson's, (1982) "Arithmetic in the Computer/Calculator Age" discussed the needs of students for learning and functioning in the coming age: algorithmic concepts, fractional comparisons, decimal approximations, understanding and estimation of percents, estimation, applications and problem solving. He encouraged teachers to change their instructional emphasis from computational to intuitive arithmetic in order to meet their students' needs.

6. Dolan and Williamson's (1983) *Teaching Problem Solving Strategies* provided problem solving strategies, curricular suggestions, and sample problems from which Kaye structured her unit on problem solving.

7. Lappan's (1983) Middle Grades Mathematics Project units on *Similarity, Probability,* and *Factors and Multiples* provided Kaye with materials and strategies which were modified and implemented as new content units for her students.

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